EXAM

Review Topics for the Final Exam

Math 5319, Spring 2010

May 5, 2010

These are review topics for the final exam, in addition to the previous sheet of review topics.

This exam has 7 problems. There are 0 points total.

Good luck!
Problem 1. Uniform Convergence.

1. Know the definition of uniform convergence.

2. Be able to give examples of sequences of functions that are uniformly convergent and pointwise convergent, but not uniformly convergent.

3. Be able to prove that if a sequence of continuous functions converges uniformly, the limit function is continuous.

4. Be able to give an example of a sequence of continuous functions that converges pointwise to a function that is not continuous.

5. Be able to state and prove the Weierstrass M-test, Theorem 7.10.

6. Be able to state Theorem 7.11.

7. Know that definition of the space of continuous functions, Definition 7.14, and the supremum norm.

8. Be able to prove the completeness of the space of continuous functions, Theorem 7.15.

9. Be able to state and prove Theorem 7.16.

10. Be able to state Theorem 7.17.

Problem 2. Equicontinuity.

1. Be able to define an equicontinuous family of functions, Definition 7.22.

2. Be able to state Theorem 7.23.

3. Be able to state and prove Theorem 7.24.

4. Be able to state the Arzela-Ascoli Theorem, Theorem 7.25.
Problem 3. The Stone-Weierstrass Theorem.

1. Be able to state the Weierstrass Approximation Theorem, Theorem 7.26.
2. Be able to define a uniformly closed algebra of functions, Definition 7.28.
3. Be able to show that the closure of an algebra is an algebra, Theorem 7.29
4. Be able to state the Real Stone-Weierstrass Theorem, Theorem 7.32. Be able to define the terms in the theorem.
5. Be able to state the Complex Stone-Weierstrass Theorem, Theorem 7.33. Be able to define the terms in the theorem.

Problem 4. Power Series.

1. Be able to define the terms power series, radius of convergence and interval of convergence. Be able to give the formula for the radius of convergence, Theorem 3.39
2. Be able to state the theorem on the product of two series, Theorem 3.50
3. Be able to state and prove Theorem 8.1.
4. Be able to state Abel’s Theorem, Theorem 8.2
5. Be able to state Theorem 8.3
6. Be able to state and prove Theorem 8.4.
7. Be able to state and prove Theorem 8.5

Problem 5. Exponential, Logarithmic and Trigonometric Functions.

1. Be able to give the series definition of the exponential function and to define the logarithm. Be able to demonstrate some of their properties, as on pages 178–182
2. Be able to give the definitions of the sine and cosine functions and derive some of their properties, as on pages 182–184
3. Be able to state the Theorem on the Algebraic Completeness of the Complex Field, Theorem 8.8

1. Be able to define the Gamma function, Definition 8.17. Be able to prove the integral converges.
2. Be able to state the properties of the Gamma function, Theorem 8.18.
3. Be able to state Theorem 8.19.
4. Be able to give the definition of the Beta function, and the formula for it in terms of the Gamma function, Theorem 8.20.

Problem 7. Functions of Several Variables.

1. Be able to define the norm of a linear transformation.
2. Be able to state and prove Theorem 9.8
3. Be able to give the definition of the derivative, Definition 9.11.
4. Be able to state and prove the Chain Rule, Theorem 9.15.
5. Be able to state Theorem 9.19.
7. Be able to state and prove the Contraction Mapping Lemma, Theorem 9.23.
9. Be able to state the Implicit Function Theorem, Theorem 9.28, defining the notation in the theorem.