• Write all of your answers on separate sheets of paper. You can keep the exam questions when you leave. You may leave when finished.

• You must show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).

• This exam has 5 problems. There are 270 points total.

Good luck!
Problem 1. In each part, find the general solution of the differential equation, or solve the initial value problem.

A. \( y'' - 3y' + 2y = 0, \quad y(0) = 1, \quad y'(0) = 3. \)

B. \( y'' + 2y' + y = 0 \)

C. \( y'' - 4y' + 13y = 0. \)

D. \( x^2y'' - 6y = 0 \)

Problem 2. Find the general solution.

A. \( (D - 1)(D - 2)(D - 3)(D + 4)y = 0 \)

B. \( D^3(D + 1)(D - 2)^3y = 0 \)

C. \( (D - 2)(D^2 - 4D + 5)^3y = 0 \)

Problem 3. Use the method of Undetermined Coefficients (either version) to find the general solution

A. \( y'' - 3y' + 2y = x^2 + 1 \)

B. \( y'' - 3y' + 2y = e^{2x} \)

Problem 4. Find the general solution by the method of variation of parameters. (No credit for doing it by a different method.)

\( x^2y'' + 3xy' - 3y = x. \)

The basic solutions are \( y_1 = x \) and \( y_2 = 1/x^3. \)
Problem 5. The motion of a weight on a spring, with no damping, is described by the equation

\[
\frac{d^2y}{dt^2} + y = 0.
\]

Suppose that the initial conditions are \( y(0) = -1 \) and \( y'(0) = 1 \).
Find the solution of the initial value problem.
Find the amplitude \( A \) of the oscillations. Write the solution in the form

\[ y = A \cos(\omega t - \varphi). \]