• This is a Takehome Exam. You may discuss the problems with others, but write up your own solutions.

• If not otherwise instructed, you can use a calculator to do the integrals, but state exactly what you used the calculator to compute.

• You must show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).

• This exam has 8 problems. There are 380 points total.

Good luck!
Problem 1. Find the Laplace Transform of the given function.

A. 
\[ f(t) = t^4 - 2t^3 + t^2 - 3t + 2 \]

B. 
\[ f(t) = 5e^{-t} + 2e^{3t} + t^2e^{5t} \]

C. 
\[ f(t) = 3e^{-2t} \sin(5t) + 4e^{-2t} \cos(5t) \]

D. 
\[ f(t) = te^{-3t} \cos(5t) \]

Problem 2. In each part, give the form of the partial fractions decomposition, with undetermined coefficients. Do not find the coefficients. No computation is required.

A. 
\[ \frac{s^2}{(s - 1)(s - 2)(s + 2)} \]

B. 
\[ \frac{s^4 + 1}{s^2(s - 1)^3(s - 3)} \]

C. 
\[ \frac{s^2 + s + 1}{(s - 1)(s^2 + 1)} \]

D. 
\[ \frac{s^9 + 5}{s^2(s^2 + 1)^3} \]
Problem 3. In each part, find the inverse Laplace Transform. (Unless otherwise stated, you can use a calculator to find the partial fractions decomposition.)

A. In this part, find the partial fractions decomposition by hand.
\[ F(s) = \frac{5s^2 - 3s + 1}{s^2(s - 1)} \]

B.
\[ F(s) = \frac{5s^4 - 15s^3 + 15s^2 - 7s + 1}{(s - 1)^3s^3} \]

C.
\[ F(s) = \frac{5s - 7}{s^2 - 6s + 13} \]

D.
\[ F(s) = \frac{s^2}{(s^2 + 1)^2} \]

Problem 4. Solve the following initial value problems by the method of Laplace Transforms.

A.
\[ y'' - 3y' + 2y = t^2 + e^t, \quad y(0) = 1, \quad y'(0) = 1 \]

B.
\[ y'' + 4y = \sin(2t), \quad y(0) = 1, \quad y'(0) = 0 \]

Problem 5. Find the Laplace Transform of the function
\[ f(t) = \begin{cases} 
  t, & 0 \leq t < 1, \\
  t^2, & 1 \leq t < 2 \\
  1, & 2 < t < \infty
\end{cases} \]
Problem 6. Let \( f(t) \) be defined by

\[
f(t) = \begin{cases} 
0, & 0 < t < 1 \\
t, & 1 < t < 2 \\
0, & 2 < t < \infty. 
\end{cases}
\]

Use Laplace Transforms to solve the initial value problem

\[ y'' + y = f(t), \quad y(0) = 0, \quad y'(0) = 1. \]

Problem 7. Find the following convolutions directly from the definition. You can use a calculator for the integrals.

A. \( t^2 \ast t^3 \)
B. \( e^{2t} \ast e^{-t} \)
C. \( \sin(t) \ast \sin(t) \).

Problem 8. In each part, use Laplace Transforms to find the convolution.

A. \( e^t \ast \cos(2t) \)
B. \( t^2 \ast \sin(3t) \)