EXAM

Exam 3
Take Home Exam

Math 2350-02, Summer II 2008

August 1, 2008

• This is a Takehome Exam. You may discuss the problems with others, but write up your own solutions.

• If not otherwise instructed, you can use a calculator to do the integrals, but state exactly what you used the calculator to compute.

• You must show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).

• This exam has 8 problems. There are 360 points total.

Good luck!
Problem 1. Find the critical points of the function

\[ f(x, y) = x^2 + y^3 + 6xy + 24y. \]

Classify each of the critical points as a relative maximum, relative minimum, or a saddle point.

Problem 2. Use Lagrange Multipliers to find the max and min of the function \( f(x, y) = xy + z \) on the sphere \( x^2 + y^2 + z^2 = 3 \).

Problem 3. Let \( D \) be the region in the \( xy \)-plane bounded by the curve \( y = 1 - x^2 \) and the line \( x + y = 1 \).

A. Find an interated integral for computing

\[ \int \int_D y \, dA \]

where the order of integration is \( dy \, dx \) (i.e., integrate first with respect to \( y \) and then with respect to \( x \)). Don’t evaluate the integral yet.

B. Find an interated integral for computing

\[ \int \int_D y \, dA \]

where the order of integration is \( dx \, dy \) (i.e., integrate first with respect to \( x \) and then with respect to \( y \)). Don’t evaluate the integral yet.

C. Evaluate one of the integrals above by hand computation.

Problem 4. Consider the iterated integral

\[ \int_0^2 \int_0^{x^2} f(x, y) \, dy \, dx. \]

Sketch the region of integration and find an equivalent integral with the order of integration reversed.

Problem 5. Let \( D \) be the region in the first quadrant bounded by the coordinate axes and the circle \( x^2 + y^2 = a^2 \). Find the centroid of \( D \), and the moment of inertia of \( D \) for rotation about the \( y \)-axis.
Problem 6. Let $D$ be the solid in the first octant bounded by the coordinate planes and the plane $2x + 2y + z = 2$.

A. Find an iterated integral for calculating the volume of $D$ where the first integration is with respect to $z$. Don’t evaluate the integral yet.

B. Find an iterated integral for calculating the volume of $D$ where the first integration is with respect to $y$. Don’t evaluate the integral yet.

C. Evaluate one of the integrals above by hand computation.

Problem 7. Let $D$ be the solid bounded by the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 2$. Use cylindrical coordinates to evaluate

$$\iiint_D x^2 z \, dV.$$ 

Problem 8. Let $D$ be the solid bounded by the sphere $x^2 + y^2 + z^2 = a^2$. Use spherical coordinates to evaluate the integral

$$\iiint_D z^2 \, dV.$$