PROBLEM SET

Practice Lagrange Multiplier Problems

Math 2350, Spring 2008

March 14, 2008

• These are practice problems (don’t turn them in) with (partial) answers provided.

Good luck!
Problem 1. Find the maximum and minimum of the function \( f(x, y) = xy^2 \) on the circle \( x^2 + y^2 = 1 \). Since the circle is a closed bounded curve, the maximum and minimum exist.

Answer: the critical points are 

\[
(x, y) = (\pm 1, 0), (x, y) = (\pm \sqrt{3}/3, \sqrt{6}/3), (x, y) = (\pm \sqrt{3}/3, -\sqrt{6}/3)
\]

Problem 2.

Find the max and min of the function \( f(x, y, z) = xy + yz \) on the sphere \( x^2 + y^2 + z^2 = 1 \). Since the sphere is a closed bounded surface, the max and min exist.

Answer: Critical points \((x, y, z)\),

\[
(1/2, \sqrt{2}, 0, -1/2 \sqrt{2}) \\
(-1/2, \sqrt{2}, 0, 1/2 \sqrt{2}) \\
(1/2, -(1/2) \sqrt{2}, 1/2) \\
(1/2, -(1/2) \sqrt{2}, 1/2) \\
(-1/2, -(1/2) \sqrt{2}, -1/2) \\
(-1/2, -(1/2) \sqrt{2}, -1/2)
\]

Problem 3.

A box is to be constructed with a volume of 1372 cubic inches. The box has 4 sides and a bottom, but no top. What are the dimensions of the cheapest box?

Answer: Height 7, other dimensions 14.

Problem 4.

Find the max and min of the function \( f(x, y) = x^2 - x/2 + y^2 - y \) on the disc enclosed by the unit circle \( x^2 + y^2 = 1 \).

Answers: There is one interior critical point at \((1/4, 1/2)\), which is the minimum. Using Lagrange multipliers, there are two critical points on the boundary
circle, namely \((\sqrt{5}/5, 2\sqrt{5})\) and \((\sqrt{5}/5, 2\sqrt{5})\). The second of these is the global maximum.

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**Problem 5.**

Let \(C\) be the curve of intersection of the cylinder \(x^2 + y^2 = 1\) and the plane \(x + y + z = 1\). Find the highest and lowest points on the curve, i.e. maximize and minimize the function \(f(x, y, z) = z\) over \(C\).

Answer: \((\sqrt{2}/2, \sqrt{2}/2, \sqrt{2})\) and \((-\sqrt{2}/2, -\sqrt{2}/2, -\sqrt{2})\)