• Write all of your answers on separate sheets of paper. You can keep the question sheet.

• You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).

• This problem set has 3 problems. There are **300 points total**.

Good luck!
Problem 1. A set $X$ is finite if there is a natural number $n$ and a bijection

$$f: \{1, 2, \ldots, n\} \to X.$$ 

In this case, the cardinality of $X$ is $n$.

If $X$ and $Y$ are finite, show that the following sets are finite:

A. $X \cap Y$
B. $X \cup Y$
C. $X \setminus Y$.

Problem 2. Suppose $X$ is finite and $A$ is infinite. Recall our proof that any infinite set has a countable subset.

A. Show $A \setminus X$ is infinite.
B. Show $|X \cup A| = |A|$.
C. If $C$ and $D$ are countable $C \cup D$ is countable.
D. If $C$ is countable then $|C \cup A| = |A|$.

Problem 3. Recall that $\mathbb{N} \times \mathbb{N}$ is countable.

1. Show that if $C$ and $D$ are countable, $C \times D$ is countable.
2. Show that for any $n \in \mathbb{N}$,

$$P_n = \underbrace{\mathbb{N} \times \mathbb{N} \times \cdots \times \mathbb{N}}_{n \text{ factors}}$$

is countable.
3. Show that

$$F = \{ S \subseteq \mathbb{N} \mid S \text{ is finite} \}$$

is countable.