This is a Take-home Exam. It is due on Tuesday, April 27.

You may look at the Textbook, your notes, and any material given out in class. You may use a calculator. You may discuss the problems with classmates, but write up your own solutions, don’t just copy somebody!

Write all of your answers on separate sheets of paper. You can keep the exam questions.

You must show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., \( \sqrt{2} \), not 1.414).

This exam has 7 problems. There are 280 points total.

Good luck!
Problem 1. Find the general solution by the method of undetermined coefficients:

\[(D^2 + 4)y = x^2 \cos(2x).\]

Problem 2. Find the Laplace Transform of the following function:

\[f(t) = \begin{cases} t, & 0 < t < 1 \\ t^2 + 5t, & 1 < t < 2 \\ 0, & 2 < t < \infty. \end{cases}\]

Problem 3. Find the Inverse Laplace Transform of the following function:

\[F(s) = \frac{1}{s} + e^{-s} \left( \frac{1}{(s-1)(s-2)} + e^{-5s} \frac{1}{s^2 + 4} \right).\]

Problem 4. Solve the following initial value problem, using Laplace Transforms:

\[y'' - 5y' + 6y = u(t-1)(2t+1), \quad y(0) = 1, \quad y'(0) = 0.\]

Problem 5. In the following problems, use formulas (1) and (6) from section 5.4.

A. Find the Laplace transform of \(f(t) = t^2 \sin(2t)\).

B. Find the Laplace transform of

\[f(t) = \frac{\cos(t) - \cos(2t)}{t}.\]

You may assume the fact that \(\lim_{t \to 0^+} f(t)\) exists.

C. Find the inverse Laplace transform of the function

\[G(s) = \ln \left( \frac{s-1}{s+1} \right).\]

Problem 6. Find the convolution \(t * e^{2t}\) directly from the definition.
Problem 7. Find the convolution $e^t \ast \cos(2t)$ using Laplace transforms.