

Formulas

Accumulation Functions:

Simple Interest: $a(t) = 1 + it$

Compound Interest: $a(t) = \left(1 + \frac{i^{(m)}}{m}\right)^{mt}$

Compound Discount: $a(t) = \left(1 - \frac{d^{(m)}}{m}\right)^{-mt}$

Force of Interest: • $\delta_t = \frac{d}{dt} \ln a(t).$

- For compound interest: $\delta = \ln(1 + i).$
- $a(t) = e^{\int_0^t \delta_r dr}.$
- For compound interest: $a(t) = e^{\delta t}.$

Accumulation and Discount Factors:

$$u = 1 + i = \frac{1}{1 - d}$$

$$v = \frac{1}{1 + i} = 1 - d$$

$$u = \frac{1}{v}.$$

Rate of Interest and Discount:

$$i = \frac{d}{v}$$

$$d = \frac{i}{u}$$

Annuities:

$$a_{\bar{n}|} = \frac{1 - v^n}{i} = \frac{v(1 - v^n)}{1 - v}$$

$$a_{\infty|} = \frac{1}{i}$$

$$\ddot{a}_{\bar{n}|} = \frac{1 - v^n}{d} = \frac{1 - v^n}{1 - v}$$

$$\ddot{a}_{\infty|} = \frac{1}{d}$$

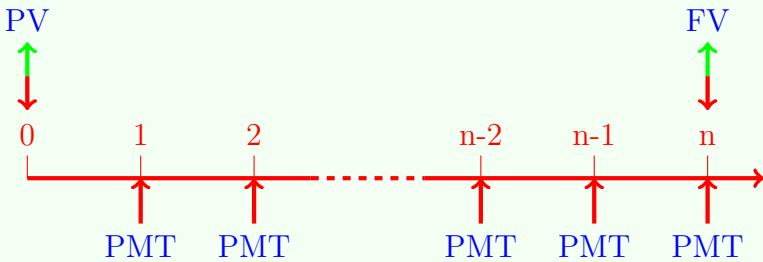
$$s_{\bar{n}|} = \frac{u^n - 1}{i} = \frac{u^n - 1}{u - 1}$$

$$\ddot{s}_{\bar{n}|} = \frac{u^n - 1}{d} = \frac{u(u^n - 1)}{u - 1}$$

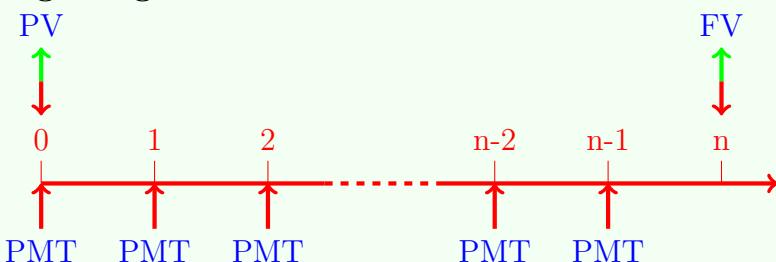
Formulas

TVM Solver:

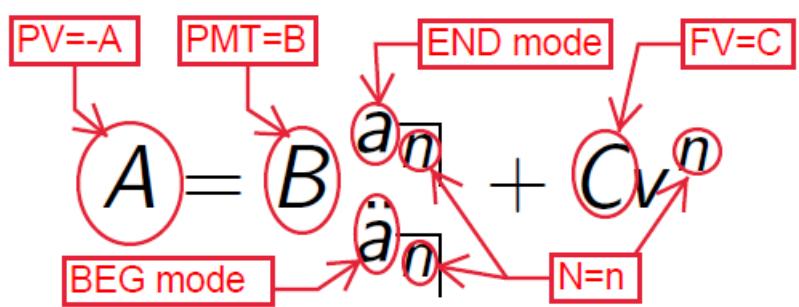
End Mode:



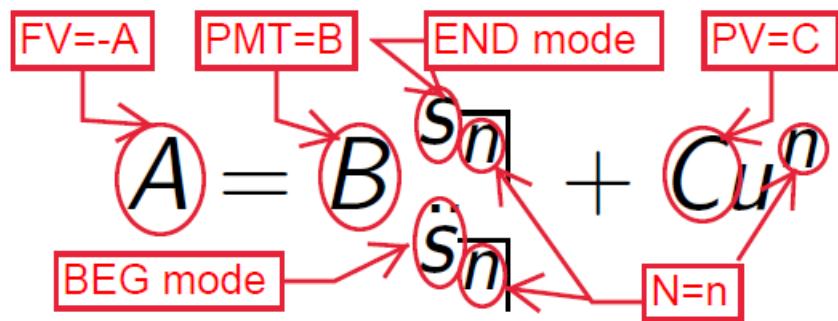
Beginning Mode:



a-angle-n:



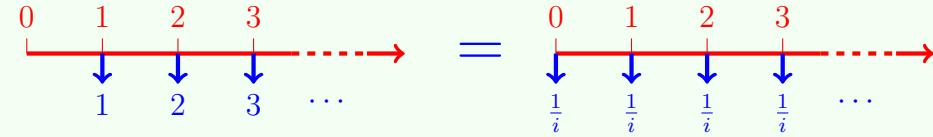
s-angle-n:



Formulas

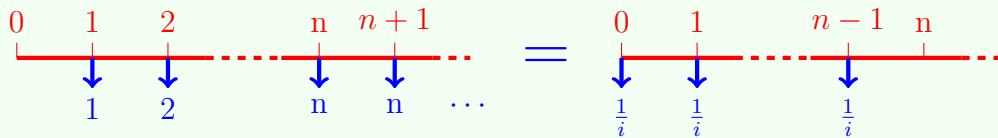
Increasing Annuities:

Perpetuity 1:



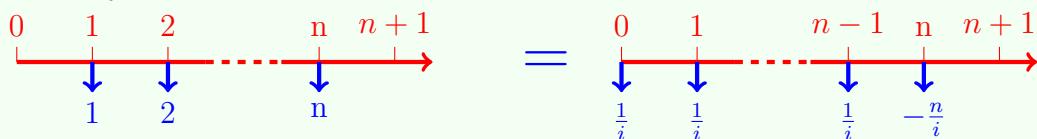
$$(Ia)_{\infty} = \frac{1}{id}$$

Perpetuity 2:



$$PV = \frac{1}{i} \ddot{a}_{\bar{n}}$$

Annuity:



$$(Ia)_{\bar{n}} = \frac{1}{i} \ddot{a}_{\bar{n}} - \frac{n}{i} v^n$$

$$(Is)_{\bar{n}} = \frac{1}{i} \ddot{s}_{\bar{n}} - \frac{n}{i}$$

Decreasing Annuities:



$$(Da)_{\bar{n}} = \frac{n}{i} - \frac{1}{i} a_{\bar{n}}$$

$$(Ds)_{\bar{n}} = \frac{n}{i} u^n - \frac{1}{i} s_{\bar{n}}$$

Continuous Annuities:

$$PV = \int_0^n PM(t) \frac{1}{a(t)} dt$$

$$FV = \int_0^n PM(t) \frac{a(n)}{a(t)} dt$$

where $PM(t)$ and $a(t)$ are the payment and accumulation functions, respectively.

Formulas

Amortization Schedules:

$$bl_k = \text{pmt } a_{\overline{n-k]} = Pu^k - \text{pmt } a_{\overline{n-k]}$$

$$pr_k = \text{pmt } v^{n-k+1} = pr_j u^{k-j} = \text{pmt} - in_k$$

$$in_k = \text{pmt}(1 - v^{n-k+1}) = bl_{k-1}i = \text{pmt} - pr_k$$

Bonds:

$$P = Fra_{\overline{n}} + Cv^n$$

$$B_j = Fra_{\overline{n-j}} + Cv^{n-j}$$

Premium Bonds:

$$P - C = C(g - i)a_{\overline{n}}$$

$$pr_k = B_{k-1} - B_k = C(g - i)a_{\overline{n}}v^{n-k+1} = pr_j u^{k-j}$$

$$in_k = Fr - pr_k$$

Discount Bonds:

$$C - P = C(i - g)a_{\overline{n}}$$

$$di_k = B_k - B_{k-1} = C(i - g)a_{\overline{n}}v^{n-k+1} = pr_j u^{k-j}$$

$$in_k = Fr + di_k$$

Forward Rates

$$f_{[t,t+m]} = \frac{u_{t+m}^{t+m}}{u_t^t} - 1.$$

Swap Rate

$$\begin{aligned} R &= \frac{\sum_{i=1}^n N_i(v_{i-1}^{t_{i-1}} - v_i^{t_i})}{\sum_{i=1}^n N_i v_i^{t_i}} \\ &= \frac{v_0^{t_0} - v_n^{t_n}}{\sum_{i=1}^n v_i^{t_i}}, \quad \text{if } N_i = N \ \forall i \\ &= \frac{1 - v_n^{t_n}}{\sum_{i=1}^n v_i^{t_i}}, \quad \text{if } N_i = N \ \forall i \text{ and } t_0 = 0 \end{aligned}$$

Formulas

Duration and Convexity

$$\text{Macaulay Duration: } d_{mac} = \frac{\sum_k t_k a_k v^{t_k}}{p(i)}$$

$$\text{Modified Duration: } d_{mod} = \frac{\sum_k t_k a_k v^{t_k+1}}{p(i)}$$

$$\text{Macaulay Convexity: } c_{mac} = \frac{\sum_k t_k^2 a_k v^{t_k}}{p(i)}$$

$$\text{Modified Convexity: } c_{mod} = \frac{\sum_k t_k(t_k + 1) a_k v^{t_k+2}}{p(i)}$$

$$\text{Macaulay Approximation } P(i) \approx P(i_0) \left(\frac{1 + i_0}{1 + i} \right)^{d_{mac}}$$

$$\text{Modified Approximation } P(i) \approx P(i_0)(1 - d_{mod}(i - i_0))$$

Redington Immunization

- 1) $A_1 v^{s_1} + \cdots + A_m v^{s_m} = L_1 v^{t_1} + \cdots + L_n v^{t_n},$
- 2) $s_1 A_1 v^{s_1} + \cdots + s_m A_m v^{s_m} = t_1 L_1 v^{t_1} + \cdots + t_n L_n v^{t_n},$
- 3) $s_1^2 A_1 v^{s_1} + \cdots + s_m^2 A_m v^{s_m} > t_1^2 L_1 v^{t_1} + \cdots + t_n^2 L_n v^{t_n}$

Full Immunization

- 1) $A u^a + B u^{-b} = L,$
- 2) $a A u^a - b B u^{-b} = 0.$