CH 7: Sampling Distributions

1. Basic Concepts

   (A) Parameter: A numerical characteristic of a population, such as a population mean μ, a population standard standard deviation σ, a population proportion p.

   (B) Sample statistic: A sample characteristic, such as sample mean $\overline{x}$, sample standard deviation s, a sample proportion $\overline{p}$.

   (C) Our goal in this chapter is to use sample statistics to estimate certain parameters, such as point estimator $\bar{x}$ for μ, point estimator $\bar{p}$ for p.

   (D) Any sample statistic will have a probability distribution called the **sampling distribution** of the statistic.

2. Sample Distribution of $\bar{x}$.

   (A) The expected value of $\bar{x}$ (or the population mean of the sample mean, denoted by $E(\bar{x})$)

   $$E(\bar{x}) = \mu \qquad \text{(eq7.1)}$$

   where μ is the population mean.

   (B) The standard deviation the sample mean (or the standard error of the sample mean, denoted by $\sigma_{\overline{x}}$)

   $$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}} \qquad \text{(eq7.3)}$$

   where σ is the population standard deviation.

   EX 1 A population has a mean of 99 and standard deviation of 7. Compute the expected value of the sample mean and the standard error of the sample mean for
   (1). $n = 4$

   (2). $n = 25$

   (C) If a random variable x is from a normal distribution, i.e., $N(\mu, \sigma)$, then the random variable sample mean $\overline{x}$ would have a normal distribution with mean μ and standard deviation $\dfrac{\sigma}{\sqrt{n}}$, i.e., $N\left(\mu, \dfrac{\sigma}{\sqrt{n}}\right)$.
   Application 1: Finding the probability of the sample mean $\overline{x}$:
   Step 1: Write down the probability statement (say: $P(\bar{x} < a)$. $P(\bar{x}) > a$, $P(a < \bar{x} < b)$)
   Step 2: Use $Z = \dfrac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$ to standardize the value of $\overline{x}$ into Z
   Step 3: Look in the standard normal table (z-table) to find the probability.

Application 2: Recovering the $\overline{x}$ value for a given probability p.

Step 1: Find the Z-value from the standard normal table for the given probability.

Step 2: Use the formula $Z = \dfrac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$ to solve for $\overline{x}$

EX 2 Apples have a mean weight of 7 ounces and a standard deviation of 2 ounces (they are normally distributed) and they are chosen at random and put in a box of 30.

(1) Find the probability that the average weight of the apples in a box is greater than 6.5 ounces.

(2) Below what value do 12.1% of the average weight of the apples fall?

(D) Question: what if the sampling is from a nonnormal population, do we have similar result? Answer: yes! if the sample size n is large enough (say, at least 30). This result is called the Central Limit Theorem: Whatever the population, the distribution of $\overline{x}$ is approximately normal with mean μ and standard deviation $\dfrac{\sigma}{\sqrt{n}}$ if n is large.

EX 3 Consider a population with mean $\mu = 82$ and standard deviation $\sigma = 12$. If a random sample of size of 64 is selected. What is the probability that the sample mean will lie between 80.8 and 83.2?

3. Sample distribution of $\bar{p}$

(A) The sample proportion $\bar{p}$ can be computed use the equation $\bar{p} = \dfrac{x}{n}$ where x is the number of elements in the sample that possess the characteristic of interest and n is the sample size.

(B) Expected value of $\bar{p}$

$$E(\bar{p}) = p \qquad \text{(eq7.4)}$$

where p is the population proportion

(C) The standard deviation of $\bar{p}$ (or called the standard error, denoted by $\sigma_{\bar{p}}$)

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} \qquad \text{(eq7.6)}$$

EX 4 A simple random sample of size 100 is selected from a population with $p = 0.40$. What is the expected value of $\bar{p}$? What is the standard error of $\bar{p}$?