

CH 6: Continuous Probability Distribution–Normal Distribution

1. Characteristics of a Normal Distribution:

(A) The distribution is bell-shaped with mean μ and standard deviation σ .

(B) The total area under the curve is 1.

(C) The empirical rule holds.

(D) The distribution function: $f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ for $-\infty < x < \infty$

(E) The probability of observing the value x between a and b :

$$P(a \leq x \leq b) = \text{Area under the curve between } a \text{ and } b$$

(F) Notation $N(\mu, \sigma)$

2. Standard Normal Distribution

(A) The standard normal distribution had a bell-shaped distribution with mean $\mu = 0$ and standard deviation $\sigma = 1$. It is denoted by $N(0, 1)$, and the random variable is denoted by Z (instead of X).

(B) We use the z -table to find the probability (area under the curve).

(C) The z -table represents the area under the curve to the left of a given value (left-tail).

Case 1 (left-tail): To find the probability $P(Z \leq a)$, we use the table directly.

EX 1 Draw the graph of standard normal and use the z -table to find $P(z \leq 1.10) =$

EX 2 Draw the graph of standard normal and use the z -table to find $P(z \leq -2.22) =$

Case 2 (right-tail): To find the probability $P(Z \geq a)$, we use the complement probability

$$P(Z \geq a) = 1 - P(Z < a)$$

EX 3 Draw the graph of standard normal and use the z -table to find $P(z \geq -0.91) =$

Case 3 (In between): To find the probability of an interval $[a, b]$, we use the formula

$$P(a \leq Z \leq b) = (\text{Area to the left of } b) - (\text{Area to the left of } a)$$

EX 4 Draw the graph of standard normal and use the z-table to find $P(-1.37 \leq z \leq 1.82)$

(D) Using the table to find a z -value if the probability is given

EX 5 Find the value of z such that the probability of being less than that value is 1.5%

EX 6 Find a value of z such that the probability of being more than that value is 30.5%

3. Application 1: Finding the probability of a random variable X that is normally distributed

Step 1: Write down the probability statement (say: $P(x < a)$, $P(x > a)$, $P(a < x < b)$.)

Step 2: Use $Z = \frac{X-\mu}{\sigma}$ (eq6.3) to cover the random variable into standard normal z :

Step 3: Look up the standard normal table to find the probability.

EX 7 Certain costs x is assumed to follow a normal distribution with a mean \$35,000 and a standard deviation of \$10,000.

(1) What is the probability that the costs will be less than \$40,000?

(2) What is the probability that the costs will be between \$45,000 and \$50,000?

4. Application 2: Find the value of x for a given probability.

Step 1: Find the z -value from the standard normal table for the given probability (left-tail or right-tail).

Step 2: Solve for x using equation $Z = \frac{X-\mu}{\sigma}$ (eq 6.3).

EX 8 Assumed that american family spends an average of \$75 with a standard deviation of \$5 on food per week (it's normally distributed). If 10.03% of the American families spend x or more on food per week. What would be the value of x ?