CH 17: Time Series Analysis and Forecasting

1. Basic Concepts:

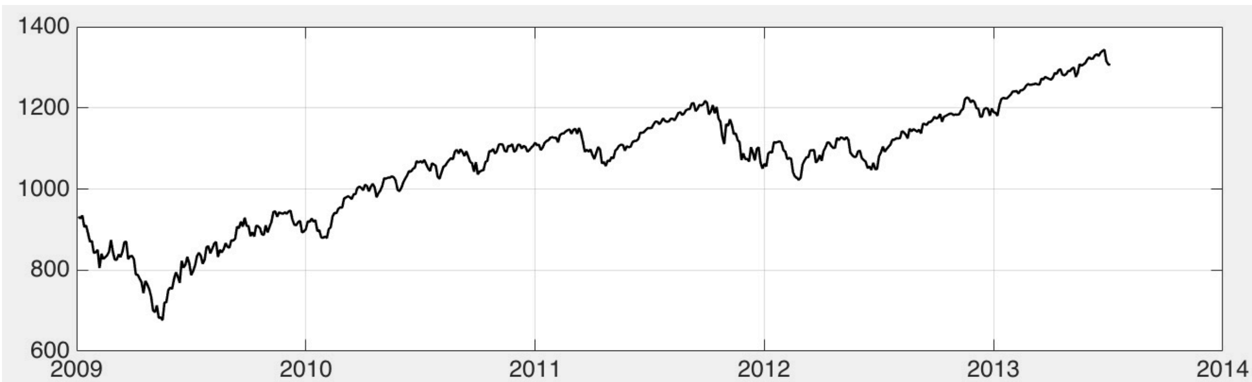
   (A) Definition: A time series is a set of observations (responses), each one being recorded at a specified time.

   (B) Plotting time-series data:

   Y-axis: Response measured at the corresponding time

   X-axis: A time indicator (in years, months and so on)

   EX 1: The S&P 500 Index for year 2009 to 2013

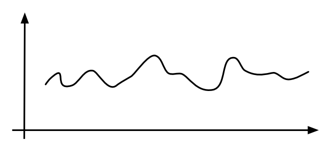


   (C) Time series analysis is used for Business Forecasting (to predict the future behavior of the estimated model). Say, you are a financial analyst and you need to forecast revenues of some companies in order to better evaluate investment opportunities for your clients.

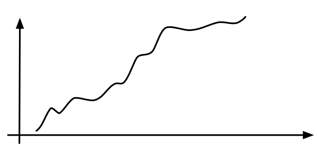
   (D) How do we do forecasting? Through identifying and isolating influence patterns of the time series.

2. Time Series Patterns

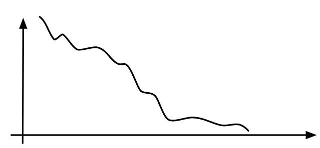
   (A) Horizontal Pattern: A horizontal pattern exists when the data fluctuate around a constant mean.



   (B) Trend Pattern: A trend is an overall or persistent, long-term upward or downward pattern of movement.

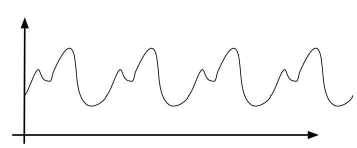


Increasing data

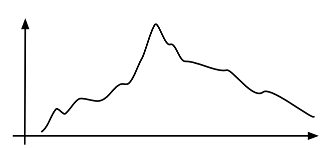


Decreasing data

   (C) Seasonal Pattern: Fairly regular periodic fluctuations that occur within some period, year after year. (Repeating patterns)

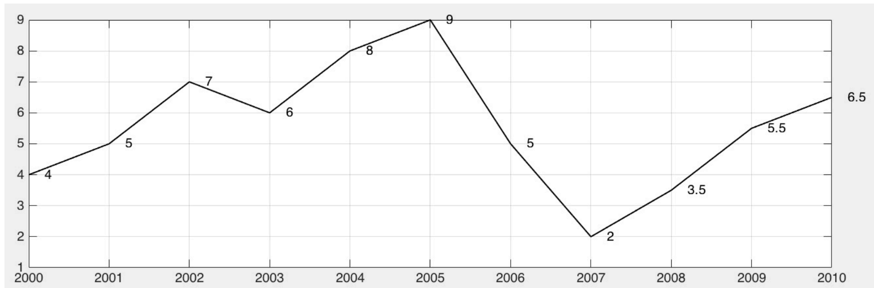


   (D) Cyclical Pattern: A wavelike pattern describing a gradual ups and downs movement that is generally apparent over a year.



3. Time-series Notation: Response: Y_t, Time $t = 1, 2, \cdots n$. Thus, at time point 1, Response is Y_1; at time point 2, Response is Y_2;..., at time point n, Response is Y_n.

EX2: The following represent total revenues (in millions) of a car rental agency over the 11-year periods from 2000 to 2010: 4.0, 5.0, 7.0, 6.0, 8.0, 9.0, 5.0, 2.0, 3.5, 5.5, and 6.5. Obtain the time-series plot.



4. Introduction to two data smoothing techniques-Moving averages and Exponential Smoothing for Forecasting.

   (A) Moving Average: The moving averages method uses the average of the most recent k data values in the time series as the forecast for the next period.

   eq17.1: Moving Average Forecast of order k:

$$F_{t+1} = \frac{\sum(\text{most recent } k \text{ data values})}{k} = \frac{y_t + y_{t-1} + ... + y_{t-k+1}}{k}$$

   where F_{t+1} is the value of the time series being computed in time period $t + 1$.

EX2 (Cont) Find the forecasting values for year 2005, 2006, and 2007 with $k = 5$.

$$F_{2005} = \tfrac{1}{5}(Y_{2004} + Y_{2003} + Y_{2002} + Y_{2001} + Y_{2000}) = \frac{8+6+7+5+4}{5} = 6$$

$$F_{2006} = \tfrac{1}{5}(Y_{2005} + Y_{2004} + Y_{2003} + Y_{2002} + Y_{2001}) = \frac{9+8+6+7+5}{5} = 7$$

$$F_{2007} = \tfrac{1}{5}(Y_{2006} + Y_{2005} + Y_{2004} + Y_{2003} + Y_{2002}) = \frac{5+9+8+6+7}{5} = 7$$

   (B) eq17.2 Exponential Smoothing Forecast

$$F_{t+1} = \alpha Y_t + (1 - \alpha) F_t$$

   (A recursive equation with $F_1 = y_1$)

   where F_{t+1} is the value of the time series being computed in time period $t + 1$, F_t is the value of the time series being computed in time period t, and α is the subjectively assigned weight or smoothing coefficient ($0 < \alpha < 1$).

EX2 (Cont) Find the forecasting values for year 2003 and 2004 using exponential smoothing technique (use $\alpha = 0.4$, $1 - \alpha = 1 - 0.4 = 0.6$)

t	y_t	Exponential Smoothing Forecasting
2000	$y_1 = 4$	$F_1 = y_1 = 4$
2001	$y_2 = 5$	$F_2 = \alpha y_1 + (1-\alpha)F_1 = \alpha y_1 + F_1 - \alpha y_1 = F_1 = 4.000$
2002	$y_3 = 7$	$F_3 = \alpha y_2 + (1-\alpha)F_2 = 0.4 \times 5 + 0.6 \times 4 = 4.400$
2003	$y_4 = 6$	$F_4 = \alpha y_3 + (1-\alpha)F_3 = 0.4 \times 7 + 0.6 \times 4.4 = 5.440$
2004	$y_5 = 8$	$F_5 = \alpha y_4 + (1-\alpha)F_4 = 0.4 \times 6 + 0.6 \times 5.44 = 5.664$