Given a function \( f(x) = x^4 - 4x^3 + 10 \).

(i) Compute the 1st & 2nd derivatives.

\[
\begin{align*}
  f'(x) &= 4x^3 - 12x^2, \\
  f''(x) &= 12x^2 - 24x \\
  &= 4x(x-3) \\
  &= 12x(x-2)
\end{align*}
\]

(ii) Draw a chart below, for the function \( f \), indicating the values of \( x \) corresponding to critical points & inflection points.

<table>
<thead>
<tr>
<th>Functions</th>
<th>Critical Numbers</th>
<th>Conclusions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = x^4 - 4x^3 + 10 )</td>
<td>( f'(x) = 0 ), ( x = 0, x = 3 )</td>
<td>1st derivative signs (increasing/decreasing)</td>
</tr>
<tr>
<td>( f(x) = 4x^2(x-3) )</td>
<td>( f''(x) = 0 ), ( x = 0, x = 2 )</td>
<td>2nd derivative signs (concavity)</td>
</tr>
<tr>
<td>( f''(x) = 12x(x-2) )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From the above table, we see there is a relative minimum at \( x = 3 \) and inflection points at \( x = 0 \) and \( x = 2 \) (because the 2nd derivative changes sign at these points), and a horizontal tangent at \( x = 0 \) (because the first derivative is zero).

(iii) Using the chart, indicate where the function \( f \) is increasing, decreasing, concave down and concave up, respectively.

\( f \) is increasing \( x \in (3, +\infty) \)

\( f \) is decreasing \( x \in (-\infty, 0) \)

Concave up \( x \in (-\infty, 0) \cup (2, +\infty) \)

Concave down \( x \in (0, 2) \)
To find the $y$-value of the critical points and the inflection points, evaluate $f$ at $x=0$, $2$, and $3$.

- $f(0) = 10$. Point of inflection $(0, 10)$
- $f(2) = -6$. Point of inflection $(2, -6)$
- $f(3) = -17$. Relative minimum at $(3, -17)$

(iv) Sketch the graph of the function $f$.

Conclude the above information.
1. Given \( f(x) = \frac{1}{3}x^3 - 9x + 2 \)

1st derivatives \( f'(x) = x^2 - 9 = (x+3)(x-3) \)
2nd derivatives \( f''(x) = 2x \).

(a) Find all critical numbers:
solve \( f'(x) = 0 \), we have \( (x+3)(x-3) = 0 \), then \( x_1 = 3, x_2 = -3 \).

(b) Find where the function is increasing and decreasing

\[
\begin{array}{c|c|c|c}
\text{Increasing} & \text{Decreasing} & \text{Increasing} \\
\text{Sign of } f'(x) & + & - & + \\
\end{array}
\]

the function \( y = f(x) \) is increasing in the interval \((-\infty, -3) \cup (3, \infty)\)
decreasing in the interval \((-3, 3)\).

(c). Find the \( x \) coordinate all points of inflection.

Solve \( f''(x) = 0 \), we have \( x = 0 \).

\( x \in (-\infty, 0) \), that is \( x < 0 \), \( f''(x) = 2x < 0 \), \( f(x) \) is Concave down
\( x \in (0, +\infty) \), that is \( x > 0 \), \( f''(x) = 2x > 0 \), \( f(x) \) is Concave up.

Therefore the inflection point is \( (0, f(0)) = (0, 2) \).

the \( x \) coordinate of inflection is \( x = 0 \).
4. Given $f(u) = 3u^4 - 2u^3 - 12u^2 + 18u - 5$.

1st derivative $f'(u) = 12u^3 - 6u^2 - 24u + 18 = 6(2u^3 - u^2 - 4u + 3)$.

$= 6(2u^3 - u^2 - 3u + 3)$

$= 6 \left[ u \ (2u^2 - 1u - 1) \right]^{2}_{-1}$

$= 6 \left[ u \ (2u^2 - 1u - 1) \right] = 6 \left[ u \ (2u - 1) \right] \ (u - 1)$

$= 6(2u + 3)(u - 1) = 6(2u + 3)(u - 1)^2$

$f''(u) = (12u^3 - 6u^2 - 24u + 18)' = 36u^2 - 12u - 24$

$= 12(3u^2 - u - 2)$

$= 12(3u + 2)(u - 1)$

(a) Determine where the function is increasing and decreasing.

Solve $f'(u) = 0$, we have $u = -\frac{3}{2}, u = 1$.

$f(x)$ is decreasing $\rightarrow$ increasing $\rightarrow$ increasing

Sign of $f'(u)$: $-\frac{3}{2}$ $+u=1$ $+$

The function $y = f(x)$ is increasing on the interval $\left( -\frac{3}{2}, +\infty \right)$

decreasing on the interval $\left( -\infty, -\frac{3}{2} \right)$

(b) Determine where the function is concave up and concave down.

Solve $f''(u) = 0$, we have $u = -\frac{3}{2}, u = 1$.

Shape of $f(x)$ is concave up $\rightarrow$ concave down $\rightarrow$ concave up.

Sign of $f''(u)$: $-\frac{3}{2}$ $-u=1$ $+$

Therefore, the function $y = f(x)$ is concave up on the interval $\left( -\infty, -\frac{3}{2} \right)$

concave down on the interval $\left( -\frac{3}{2}, 1 \right)$.

The inflection points are $\left( -\frac{3}{2}, f(-\frac{3}{2}) \right)$ and $\left( 1, f(1) \right)$. 