Math 1352-11 — WW08 Solutions

November 19, 2008

Assigned problems: 8.3 – ww_1; 8.4 – ww_2; 8.5 – 4, 6, 26, 44; 8.6 – ww_7, ww_8, 34, ww_10, 50

Always read through the solution sets even if your answer was correct. Note that like many of the integrals in this course, there is frequently more than one way to determine convergence or divergence of a series. Your solution may be correct even if you used a different method than what I use here.

1. (Note, webwork creates slight variations on this problem for each student. So your problem may be slightly different.)

$$\begin{aligned}
\int_1^\infty 3x^2 e^{-x^3}\,dx &= \lim_{N\to\infty} \int_1^N 3x^2 e^{-x^3}\,dx \\
&\quad \textit{We'll do a simple u-substitution. Let } u = x^3,\ du = 3x^2\,dx\text{:} \\
&= \lim_{N\to\infty} \int_{x=1}^{x=N} e^{-u}\,du \\
&= \lim_{N\to\infty} -e^{-u}\Big|_{x=1}^{x=N} \\
&= \lim_{N\to\infty} -e^{-x^3}\Big|_1^N \\
&= \lim_{N\to\infty} -e^{-N^3} + e^{-1} \\
&= \lim_{N\to\infty} \left(\frac{1}{e} - \frac{1}{e^{N^e}}\right) \\
&= \boxed{\frac{1}{e}}
\end{aligned}$$

So the improper integral converges to $1/e$. Next, we are asked to determine whether the following series converges.

$$\sum_{n=1}^{\infty} 3n^2 e^{-n^3}$$

Given the integral we just did, it's a rather big hint that we might want to try the **integral test**. In order to apply the integral test, let $f(x) = 3x^2 e^{-x^3}$. The integral test applies if $f(x)$ is positive, continuous, and decreasing. So first we need to check that we've met the conditions of the test. The function $f(x)$ is clearly positive since x^2 is positive and e to any power is also positive. It is continuous since it's the product of continuous functions (known from Calc. I). So the only thing left to show is that $f(x)$ is decreasing. A function is decreasing if its derivative is negative.

$$\begin{aligned}
f(x) &= 3x^2 e^{-x^3} \\
f'(x) &= 6xe^{-x^3} + (3x^2)(e^{-x^3})(-3x^2) \quad \textit{(Product rule and chain rule.)} \\
&= \frac{6x - 9x^4}{e^{x^3}}
\end{aligned}$$

The denominator is always positive. The numerator is negative for all $x > \sqrt[3]{2/3}$. So $f(x)$ is "eventually decreasing." Therefore, the integral test applies, and the series and the improper integral either both converge or both diverge. The integral converges, so $\boxed{\textbf{the series converges}}$.

2. (Note that webwork creates random, but similar, problems for each student on this question. So the solutions here are for a sample of problems to give you some examples. If you have questions on any particular problem you were given, please ask.

(2.1) $\sum_{n=1}^{\infty} \frac{7n^6}{n^8+3} = \sum_{n=1}^{\infty} a_n$: Since the series looks almost like a p-series (except for the +3 in the denominator it looks like a p-series with $p = 2$), I'll try **limit comparison test**. The given a_n terms are "similar to" $b_n = \frac{7n^6}{n^8}$.

$$\begin{aligned}
\lim_{n\to\infty} \frac{a_n}{b_n} &= \lim_{n\to\infty} \frac{7n^6/(n^8+3)}{7n^6/n^8} \\
&= \lim_{n\to\infty} \frac{\cancel{7n^6} \cdot n^8}{\cancel{7n^6} \cdot (n^8+3)} \\
&= \lim_{n\to\infty} \frac{1}{1+3/n^8} \\
&= 1
\end{aligned}$$

The limit is positive and finite (i.e., $0 < 1 < \infty$) therefore the a_n and b_n series converge or diverge together.

$$\sum_{n=1}^{\infty} \frac{7n^6}{n^8} = \sum_{n=1}^{\infty} \frac{7}{n^2}$$

is a convergent p-series ($p = 2 > 1$), therefore converges. So the original series also $\boxed{\textbf{converges}}$ by the limit comparison test.

(2.2) $\sum_{n=1}^{\infty} \frac{7n^8-n^7+7\sqrt{n}}{4n^{10}-n^6+3} = \sum_{n=1}^{\infty} a_n$: Trying the **limit comparison test**. The given a_n terms are "similar to" $b_n = \frac{7n^8}{4n^{10}}$.

$$\begin{aligned}
\lim_{n\to\infty} \frac{a_n}{b_n} &= \lim_{n\to\infty} \frac{(7n^8-n^7+7\sqrt{n})(4n^{10})}{(4n^{10}-n^6+3)(7n^8)} \\
&= \lim_{n\to\infty} \frac{(28n^{10}-4n^9+28n^{5/2})}{(28n^{10}-7n^6+21)} \\
&= \lim_{n\to\infty} \frac{(28-4/n+28/n^{2/15})}{(28-7/n^4+21/n^{10})} \\
&= 1
\end{aligned}$$

The limit is positive and finite, so $\sum a_n$ and $\sum b_n$ converge or diverge together.

$$\sum b_n = \sum \frac{7}{4} \cdot \frac{1}{n^2}$$

is a convergent p-series ($p = 2 > 1$). Therefore the original series also $\boxed{\textbf{converges}}$.

(2.3) $\sum_{n=1}^{\infty} \frac{(\ln n)^3}{n+3} = \sum_{n=1}^{\infty} a_n$: The numerator looks pretty large compared to the denominator, so I would try the divergence test first. In this case $\lim_{n\to\infty} a_n = 0$, so the divergence test doesn't tell us anything.

Next I'll try a limit comparison with $\sum b_n = \sum \frac{1}{n}$, which is a divergent p-series ($p = 1$).

$$\begin{aligned}
\lim_{n\to\infty} \frac{a_n}{b_n} &= \lim_{n\to\infty} \frac{(\ln n)^3/(n+3)}{1/n} \\
&= \lim_{n\to\infty} \frac{n(\ln n)^3}{n+3} \\
&= \lim_{n\to\infty} \frac{(\ln n)^3 + 3n(\ln n)^2(1/n)}{1} \\
&= \lim_{n\to\infty} (\ln n)^3 + 3(\ln n)^2 \\
&= \infty
\end{aligned}$$

The limit goes to infinity, so the limit comparison test doesn't apply. However, the zero-infinity version of the limit comparison test does. The limit of $a_n/b_n = \infty$ and $\sum b_n$ diverges, therefore $\sum a_n$ also $\boxed{\textbf{diverges}}$.

(2.4) $\sum_{n=1}^{\infty} \frac{7n^6}{n^7+3} = \sum_{n=1}^{\infty} a_n$: This series looks "similar" to $\sum b_n = \sum 7n^6/n^7 = \sum 7/n$, which is a divergent p-series. So I'll compare with that.

$$\begin{aligned}
\lim_{n\to\infty} \frac{a_n}{b_n} &= \lim_{n\to\infty} \frac{7n^6/(n^7+3)}{7/n} \\
&= \lim_{n\to\infty} \frac{n^7}{n^7+3} \\
&= \lim_{n\to\infty} \frac{1}{1+3/n^7} \\
&= 1
\end{aligned}$$

The limit is positive and finite, so by the **limit comparison test**, $\sum a_k$ and $\sum b_k$ converge or diverge together. $\sum b_k$ is a divergent p-series, so $\sum a_k$ $\boxed{\textbf{diverges}}$.

(2.5) $\sum_{n=1}^{\infty} \frac{\cos^2(n)\sqrt{(n)}}{n^6} = \sum_{n=1}^{\infty} a_n$. Note that $0 \le \cos^2 n \le 1$. Therefore,

$$0 \le \frac{\cos^2(n)\sqrt{(n)}}{n^6} \le \frac{\sqrt{n}}{n^6} \le \frac{1}{n^{11/2}}$$

So $0 \le a_k \le b_k$ and $\sum b_k$ is a convergence p-series ($p = 11/2 > 1$). Therefore by the **direct comparison test**, $\sum a_k$ $\boxed{\textbf{converges}}$.

3. (8.5 #4) $\sum_{n=1}^{\infty} \frac{k!}{2^k}$

   I almost always try the **ratio test** when there is a factorial. So I'll try that.

$$\begin{aligned}
\lim_{n\to\infty} \frac{a_{n+1}}{a_n} &= \lim_{n\to\infty} \frac{(n+1)!/2^{n+1}}{n!/2^n} \\
&= \lim_{n\to\infty} \frac{2^n (n+1)!}{2^{n+1} n!} \\
& \quad \textit{(Notice that } (n+1)! = (n+1)n! \textit{ and } 2^{n+1} = 2^n 2^1 \textit{)} \\
&= \lim_{n\to\infty} \frac{\cancel{2^n}(n+1)\cancel{n!}}{2 \cdot \cancel{2^n}\cancel{n!}} \\
&= \lim_{n\to\infty} \frac{n+1}{2} \quad = \quad \infty
\end{aligned}$$

   According to the ratio test, if the limit is > 1 (including ∞), the series diverges. So this series $\boxed{\textbf{diverges}}$.

4. (8.5 #6) $\sum_{n=1}^{\infty} \frac{3^k}{k!}$

   Another factorial, so I'll try the **ratio test** again.

$$\begin{aligned}
\lim_{k\to\infty} \frac{a_{k+1}}{a_k} &= \lim_{k\to\infty} \frac{3^{k+1}/(k+1)!}{3^k/k!} \\
&= \lim_{k\to\infty} \frac{3^{k+1} k!}{3^k (k+1)!} \\
&= \lim_{k\to\infty} \frac{3^k \cdot 3 \cdot k!}{3^k (k+1) k!} \\
&= \lim_{k\to\infty} \frac{\cancel{3^k} \cdot 3 \cdot \cancel{k!}}{\cancel{3^k}(k+1)\cancel{k!}} \\
&= \lim_{k\to\infty} \frac{3}{k+1} \quad = \quad 0
\end{aligned}$$

   The limit is less that 1, so the series $\boxed{\textbf{converges}}$ by the ratio test.

5. (8.5 #26) $\sum_{n=1}^{\infty} \left(\frac{k}{2k+1}\right)^k$

   Since the terms are all to the k^{th} power, I'll try the **root test**.

$$\begin{aligned}
\lim_{k\to\infty} \sqrt[k]{a_k} &= \lim_{k\to\infty} \sqrt[k]{\left(\frac{k}{2k+1}\right)^k} \\
&= \lim_{k\to\infty} \left(\frac{k}{2k+1}\right) \\
&= \lim_{k\to\infty} \frac{1}{2+1/k} \quad = \quad 1/2
\end{aligned}$$

   The limit is less than 1, so the series $\boxed{\textbf{converges}}$ by the root test.

6. (8.5 #44) $\sum_{n=1}^{\infty} \frac{1}{(\ln k)^k}$

   Again, terms are to the k^{th} power, so I'll try the **root test**.

$$\begin{aligned} \lim_{k\to\infty} \sqrt[k]{a_k} &= \lim_{k\to\infty} \sqrt[k]{\frac{1}{(\ln k)^k}} \\ &= \lim_{k\to\infty} \frac{1}{\ln k} = 0 \end{aligned}$$

   The limit is 0, which is less that 1, so the series $\boxed{\textbf{converges}}$.

7. (Note that webwork creates random, but similar, problems for each student on this question. So the solutions here are for a sample of problems to give you some examples. If you have questions on any particular problem you were given, please ask.

   (7.1) $\sum_{n=1}^{\infty} \frac{(-1)^n e^n}{n!}$:

   The is an alternating series, so we'll try the **alternating series test**. So we need to show that the limit of the terms a_n goes to zero and that the terms are decreasing.

   We can show that $\lim_{k\to\infty} a_k = 0$ using the squeeze theorem (from calc 1).

$$\begin{aligned} 0 \le \frac{e^n}{n!} &= \frac{e^n}{n(n-1)(n-2)\cdots(3)(2)(1)} \\ &= \frac{e^n}{[n(n-1)(n-2)\cdots(\frac{n}{2})][(\frac{n}{2}-1)(\frac{n}{2}-2)\cdots(3)(2)(1)]} \\ & \quad \textit{(Assuming that n is even, I've just split the factorial in half at the term n/2.)} \\ &= \frac{e^n}{[n(n-1)(n-2)\cdots(\frac{n}{2})][(\frac{n}{2}-1)!]} \\ &\le \frac{1}{\left(\frac{n}{2}-1\right)!} \cdot \frac{e^n}{\left(\frac{n}{2}\right)^{n/2}} \\ &= \frac{1}{\left(\frac{n}{2}-1\right)!} \cdot e^{n-\frac{n}{2}\ln(n/2)} \\ &\le \frac{1}{\left(\frac{n}{2}-1\right)!} \\ & \quad \textit{(which goes to 0 as } n\to\infty\textit{)} \end{aligned}$$

   We could make a similar argument for odd n. Therefore, the a_n terms are squeezed between two sequences that go to zero, so $\lim_{n\to\infty} a_n = 0$.

(Note that the part above is messier than I would give on an exam). Now we need to show that $a_{n+1} \leq a_n$.

$$\begin{aligned}
\frac{e^{n+1}}{(n+1)!} &\leq \frac{e^n}{n!} \text{ ?} \\
\frac{n!e^{n+1}}{(n+1)!e^n} &\leq 1 \text{ ?} \\
\frac{e}{(n+1)} &\leq 1 \text{ ?} \\
e &\leq n+1\text{?} \\
& \quad \textit{Yes, this is true for all } n > 2.
\end{aligned}$$

Therefore this alternating series $\boxed{\textbf{converges}}$.

(7.2) $\sum_{n=1}^{\infty} \frac{(-1)^n n^n}{n!}$:
First we'll check if the terms of the series go to zero.

$$\lim_{k\to\infty} \frac{n^n}{n!} = \infty$$

(See Ex. 6 on p. 499 of the Strauss text for this limit.) We know from this that the series diverges, since it fails the divergence test. However, the alternating series test only applies if this limit is zero, so the webwork answer is "N".

(7.3) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+5}$:
First we'll check if the terms of the series go to zero.

$$\lim_{k\to\infty} \frac{1}{n^2+5} = 0$$

So the first condition of the alternating series test is met. Next we need to check if $a_{n+1} \leq a_n$.

$$\begin{aligned}
(n+1)^2 &> n^2 \\
(n+1)^2+1 &> n^2+1 \\
\frac{1}{(n+1)^2+5} &< \frac{1}{n^2+5} \\
a_{n+1} &< a_n
\end{aligned}$$

Both conditions of the alternating series test are met, so the series $\boxed{\textbf{converges}}$.

(7.4) $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^n}$:
First we'll check if the terms of the series go to zero.

$$\lim_{k\to\infty} \frac{n!}{n^n} = 0$$

(See Ex. 6 on p. 499 of the Strauss text for this limit.)
Next we need to check if $a_{n+1} \leq a_n$.

$$\begin{aligned}
\frac{(n+1)!}{(n+1)^{n+1}} &= \frac{(n+1)n!}{(n+1)^n \cdot (n+1)} \\
&= \frac{n!}{(n+1)^n} \\
&\leq \frac{n!}{n^n}
\end{aligned}$$

So $a_{n+1} \leq a_n$. Both conditions of the alternating series are met, so the series $\boxed{\textbf{converges}}$.

(7.5) $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^n}$:
The limit of $a_k \neq= 0$ (which we know because it is the reciprocal of the limit in (7.1)). So the alternating series test does not apply and the webwork answer is "N".

Note, however, that the limit not being zero means that the series diverges by the divergence test.

(7.6) $\sum_{n=1}^{\infty} \frac{(-1)^n \cos(n\pi)}{n^5}$:
Note that $\cos(n\pi)$ is -1 for odd n and 1 for even n. Since $(-1)^n)$ is also -1 for odd n and 1 for even n, the terms of the series are always positive. So this is not an alternating series, and the alternating series test does not apply.

8. (Note that webwork creates random, but similar, problems for each student on this question. So the solutions here are for a sample of problems to give you some examples. If you have questions on any particular problem you were given, please ask.

(8.1) $\sum_{n=1}^{\infty} \frac{(-1)^{k+1} k}{k^2+1}$:
We'll check first if the series converges absolutely, so we'll look at the convergence of

$$\sum |a_k| = \sum \frac{k}{k^2+1}$$

This is very similar to the divergent p-series $\sum \frac{1}{k}$, so we'll do a limit comparison with that series.

$$\begin{aligned} \lim_{k\to\infty} \frac{k/(k^2+1)}{1/k} &= \lim_{k\to\infty} \frac{k^2}{k^2+1} \\ &= 1 \end{aligned}$$

The limit is positive and finite, so by the **limit comparison test** the two series converge or diverge together. So the series $\boxed{\textbf{diverges}}$.

The series is not absolutely convergent, so we still need to check conditional convergence.

Since it's an alternating series, we'll try the **alternating series test**. So we need to check that the $\lim_{n\to\infty} a_k = 0$ and $a_{k+1} \leq a_k$.

$$\begin{aligned} \lim_{k\to\infty} a_k &= \lim_{k\to\infty} \frac{k}{k^2+1} \\ &= \lim_{k\to\infty} \frac{1}{2k} \quad (L'hopital) \\ &= 0 \end{aligned}$$

Now we just need to check that $a_{k+1} \leq a_k$.

$$\begin{aligned}
\textit{Is } \frac{k+1}{(k+1)^2+1} &\leq \frac{k}{k^2+1} \quad ? \\
\frac{(k+1)/((k+1)^2+1)}{k/(k^2+1)} &\leq 1 \quad ? \\
\frac{(k+1)(k^2+1)}{k(k+1)^2+1)} &\leq 1 \quad ? \\
\frac{k^3+k^2+k+1}{k^3+2k^2+2k} &\leq 1 \quad ? \\
k^3+k^2+k+1 &\leq k^3+2k^2+2k \quad ? \\
k^2+k+1 &\leq 2k^2+2k \quad ? \\
1 &\leq k^2+k \quad ?
\end{aligned}$$

This is true for all $k \geq 1$.

The conditions of the alternating series test are met, so this series converges. It does not converge absolutely, so it $\boxed{\textbf{converges conditionally}}$.

(8.2) $\sum_{n=1}^{\infty} \frac{(-1)^{k+1}k^2}{e^k}$:
I'll check absolute convergence first using the **generalized ratio test**.

$$\begin{aligned}
\lim_{k\to\infty} \left|\frac{a_{k+1}}{a_k}\right| &= \lim_{k\to\infty} \left|\frac{(-1)^{k+2}(k+1)^2/e^{k+1}}{(-1)^{k+1}(k^2/e^k}\right| \\
&= \lim_{k\to\infty} \frac{e^k(k+1)^2}{e^{k+1}k^2} \\
&= \lim_{k\to\infty} \frac{(k+1)^2}{ek^2} \\
&= \lim_{k\to\infty} \frac{(k^2+2k+1}{ek^2} \\
&= \lim_{k\to\infty} \frac{(1+2/k+1/k^2}{e} \\
&= 1/e
\end{aligned}$$

The limit is < 1, so by the generalized ratio test, the series $\boxed{\textbf{converges absolutely}}$.

(8.3) $\sum_{n=1}^{\infty} \frac{(-1)^{k+1}k}{2k+1}$:
This series looks like it might fail the **divergence test**, so I will try that first.

$$\begin{aligned}
\lim_{k\to\infty} a_k &= \lim_{k\to\infty} \frac{k}{2k+1} \\
&= \lim_{k\to\infty} \frac{1}{2+1/k} \\
&= 1/2
\end{aligned}$$

The limit $\lim_{k\to\infty} a_k \neq 0$ therefore the series **must diverge**.

(8.4) $\sum_{n=1}^{\infty} \frac{(-1)^{k+1}}{k^{3/2}}$:

This series is an **alternating p-series** with $p = \frac{3}{2} > 0$. We proved in class that alternating p-series converge for $p > 0$, so it converges at least conditionally.

To determine absolute convergence, we look at

$$\sum \left| \frac{(-1)^{k+1}}{k^{3/2}} \right| = \sum \frac{1}{k^{3/2}}$$

This is a p-series with $p = 3/2$. A p-series with $p > 1$ converges, therefore this series converges. This means that the original series $\boxed{\textbf{converges absolutely}}$.

(8.5) $\sum_{n=1}^{\infty} \frac{(-1)^{k+1} k!}{\ln k}$:

The factorial always makes me think to try the ratio test. So I'll test for absolute converges first with the **generalize ratio test**.

$$\begin{aligned}
\lim_{k\to\infty} \left| \frac{a_{k+1}}{a_k} \right| &= \lim_{k\to\infty} \left| \frac{(-1)^{k+1}\ (k+1)!\ /\ \ln(k+1)}{(-1)^k\ k!\ /\ \ln k} \right| \\
&= \lim_{k\to\infty} \frac{(k+1)!\ /\ \ln(k+1)}{k!/\ln k} \\
&= \lim_{k\to\infty} \frac{\ln k\ (k+1)!}{\ln(k+1)\ k!} \\
&= \lim_{k\to\infty} \frac{\ln(k)\ (k+1)}{\ln(k+1)} \\
&= \lim_{k\to\infty} \frac{\frac{1}{k}(k+1) + \ln(k)}{\frac{1}{k+1}} \quad \textit{(L'hopital)} \\
&= \lim_{k\to\infty} \left[\frac{(k+1)^2}{k} + (k+1)\ln k \right] \\
&= \infty
\end{aligned}$$

The limit is > 1, therefore according to the generalized ratio test, the series $\boxed{\textbf{diverges}}$.

9. (8.6 #34)

(a.) The first for terms S_4 of the alternating series give

$$\begin{aligned}
S_4 &= \frac{-1}{1} + \frac{1}{4} - \frac{1}{9} + \frac{1}{16} \\
&= \frac{-115}{144} \\
&\approx \boxed{-0.7986}
\end{aligned}$$

(b.) The error in this estimate from taking 4 terms is

$$|S - S_4| \leq |a_5| = \left| \frac{-1}{25} \right| = \frac{1}{25}$$

So the error is $\boxed{\leq 0.04}$.

(c.) How many terms for 3-place accuracy? We need

$$\begin{aligned}
|S - S_N| &\leq |a_{N+1}| < 0.0005 \\
\frac{1}{(N+1)^2} &\leq \frac{1}{2000} \\
(N+1)^2 &\geq 2000 \\
N &\geq \sqrt{2000} - 1 \\
&\approx 43.72
\end{aligned}$$

So we need $N \geq 44$ or at least $\boxed{\textbf{44 terms}}$ of the series for 3 digits of accuracy.

10. This series is the alternating series

$$1 - \frac{1}{10} + \frac{1}{100} - \frac{1}{1000} + \cdots \quad = \quad \sum_{k=0}^{\infty} \frac{(-1)^k}{10^k}$$

So we need

$$\begin{aligned}
|S - S_N| &\leq |a_{N+1} < 0.001 \\
\frac{1}{10^{N+1}} &\leq \frac{1}{10^3} \\
10^{N+1} &\geq 10^3 \\
N + 1 &\geq 3 \\
N &\geq 2
\end{aligned}$$

So for this accuracy we need $N \geq 2$ or at least S_2. Note that the series starts with $N = 0$ so S_2 is the first $\boxed{\textbf{3 terms}}$ of the series.

11. Find all $x \geq 0$ such that $\sum_{k=1}^{\infty} \frac{x^{2k}}{k}$ converges:

First we'll notice that the series converges (trivially) for $x = 0$ since each term is identically 0. For $x \neq 0$, we'll try the **generalized ratio test**:

$$\begin{aligned}
\lim_{k\to\infty} \left| \frac{a_{k+1}}{a_k} \right| &= \lim_{k\to\infty} \left| \frac{x^{2(k+1)} \;/\; (k+1)}{x^{2k} \;/\; k} \right| \\
&= \lim_{k\to\infty} \left| \frac{k\, x^{2k+2}}{(k+1)\, x^{2k}} \right| \\
&= \lim_{k\to\infty} \left(\frac{k}{k+1} \right) |x^2| \\
&= \lim_{k\to\infty} \left(\frac{1}{1 + 1/k} \right)^{1} |x^2| \\
&= \lim_{k\to\infty} |x^2| \;=\; L
\end{aligned}$$

According to the generalized ratio test, the series converges for $L < 1$ and diverges for $L > 1$. So this series converges for $x^2 = L < 1$, i.e., for $x < 1$. It diverges when $x^2 > 1$, i.e., when $x > 1$ (since we are only asked about nonnegative values of x.

The test is inconclusive at $L = 1$, so we need to check the end point $x = 1$ separately. (If we were considering negative values of x, too, we would also have to check $x = -1$.) When $x = 1$ we have

$$\begin{aligned} \sum_{k=1}^{\infty} \frac{x^{2k}}{k} &= \sum_{k=1}^{\infty} \frac{1^{2k}}{k} \\ &= \sum_{k=1}^{\infty} \frac{1}{k} \end{aligned}$$

which is a divergent p-series ($p = 1$). Therefore, the series diverges for $x = 1$.

To summarize, the $\boxed{\textbf{series converges absolutely for } 0 \leq x < 1 \textbf{ and diverges for } x \geq 1}$.