Assigned problems: 7.1 — 2, 4, 8, 12, 28; 7.2 — 2, 4, 8, 16, 22, 26

Always read through the solution sets even if your answer was correct.

1. (7.1 #2)

\[
\int \frac{\ln x}{x} \, dx
\]

Let \( u = \ln x \), then \( du = \frac{1}{x} \, dx \), and we rewrite the integral as:

\[
\int \frac{\ln x}{x} \, dx = \int u \, du = \frac{1}{2} u^2 + C = \frac{1}{2} (\ln x)^2 + C
\]

2. (7.1 # 4)

\[
\int \cos(x)e^{\sin(x)} \, dx
\]

Let \( u = \sin x \), then \( du = \cos x \, dx \), and we can write the integral as:

\[
\int \cos(x)e^{\sin(x)} \, dx = \int e^u \, du = e^u + C = e^{\sin x} + C
\]

3. (7.1 # 8)

\[
\int \frac{4x^3 - 4x}{x^4 - 2x^2 + 3} \, dx
\]

Let \( u = x^4 - 2x^2 + 3 \), then \( du = 4x^3 - 4x \, dx \), and we can write the integral as:

\[
\int \frac{4x^3 - 4x}{x^4 - 2x^2 + 3} \, dx = \int \frac{du}{u} = \ln u + C = \ln(x^4 - 2x^2 + 3) + C
\]
4. (7.1 # 12)

\[ \int \frac{(2x-1)}{(4x^2-4x)^2} \, dx \]

First we’ll factor the denominator as \((4x^2-4x)^2 = [4(x^2-x)]^2 = 16(x^2-x)^2\) giving us

\[ \frac{1}{16} \int \frac{(2x-1)}{(x^2-x)^2} \, dx \]

Now we let \(u = x^2 - x\), then \(du = 2x - 1\), and we can write the integral as

\[ \frac{1}{16} \int \frac{du}{u^2} \]

\[ = -\frac{1}{16}u^{-1} + C \]

\[ = -\frac{1}{16(x^2-x)} + C \]

5. (7.1 # 28)

\[ \int x\sqrt{x+1} \, dx \]

Let \(u = x + 1\), then \(du = dx\) (and note that \(x = u - 1\)). Then we can write the integral as

\[ \int x\sqrt{x+1} \, dx = \int (u-1)u^{1/2} \, du \]

\[ = \int u^{3/2} - u^{1/2} \, du \]

\[ = \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C \]

\[ = \frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C \]

6. (7.2 # 2)

\[ \int x \sin x \, dx \]

Let \(u = x\) and \(dv = \sin x \, dx\). Then \(du = dx\) and \(v = -\cos x\). Integration by parts gives us:

\[ \int x \sin x \, dx = \int u \, dv \]

\[ = uv - \int v \, du \]

\[ = -x \cos x + \int \cos x \, dx \]

\[ = -x \cos x + \sin x + C \]

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7. (7.2 # 4)

\[ \int x \tan^{-1} x \, dx \]

Let \( u = \tan^{-1} x \) and \( dv = dx \). Then \( du = \frac{1}{x^2+1} \, dx \) and \( v = \frac{1}{2}x^2 \). Integration by parts gives us:

\[
\int x \tan^{-1} x \, dx = \int u \, dv = uv \int v \, du \\
= \frac{1}{2}x^2 \tan^{-1} x - \frac{1}{2} \int \frac{x^2}{1+x^2} \, dx \\
(\text{Use integration table for this integral. Formula 57, Appendix D.}) \\
= \frac{1}{2}x^2 \tan^{-1} x - \frac{1}{2} (x - \tan^{-1} x) + C
\]

8. (7.2 # 8)

\[ \int e^{2x} \sin(3x) \, dx \]

Let \( u = e^{2x} \) and \( dv = \sin(3x) \, dx \). Then \( du = 2e^{2x} \, dx \) and \( v = -\frac{1}{3} \cos(3x) \). Integration by parts gives us:

\[
\int e^{2x} \sin(3x) \, dx = \int u \, dv = uv - \int v \, du \\
= -\frac{1}{3} e^{2x} \cos(3x) + \frac{2}{3} \int e^{2x} \cos(3x) \, dx
\]

We still can’t solve the new integral, but it looks like if we try integration by parts again, we’ll get back the original integral.

Let \( u = e^{2x} \) and \( dv = \cos(3x) \, dx \). Then \( du = 2e^{2x} \, dx \) and \( v = \frac{1}{3} \sin(3x) \) and we’ll get:

\[
\int e^{2x} \sin(3x) \, dx = -\frac{1}{3} e^{2x} \cos(3x) + \frac{2}{3} \left( \frac{1}{3} e^{2x} \sin(3x) - \frac{2}{3} \int e^{2x} \sin(3x) \, dx \right) \\
= -\frac{1}{3} e^{2x} \cos(3x) + \frac{2}{9} e^{2x} \sin(3x) - \frac{4}{9} \int e^{2x} \sin(3x) \, dx
\]

Notice that the new integral on the right (\( \int v \, du \)) is equal to \(-\frac{4}{9}\) times our original integral. We can move this integral
to the left-hand side and solve for the value of the integral:

\[
\begin{align*}
\left(1 + \frac{4}{9}\right) \int e^{2x} \sin(3x) \, dx &= -\frac{1}{3} e^{2x} \cos(3x) + \frac{2}{9} e^{2x} \sin(3x) \\
\frac{13}{9} \int e^{2x} \sin(3x) \, dx &= -\frac{1}{3} e^{2x} \cos(3x) + \frac{2}{9} e^{2x} \sin(3x) \\
\int e^{2x} \sin(3x) \, dx &= -\frac{3}{13} e^{2x} \cos(3x) + \frac{2}{13} e^{2x} \sin(3x) + C
\end{align*}
\]

9. (7.2 # 16)

\[\int \frac{\ln(\sin x)}{\tan x} \, dx\]

There are multiple ways to do this integral, but I found substitution to be easiest. Let \( u = \ln(\sin x) \), then using the Chain Rule, \( du = \frac{1}{\sin x} \cdot \cos x \, dx = \frac{\cos x}{\sin x} \, dx = \frac{1}{\tan x} \, dx \). So we can rewrite the integral as:

\[
\int \frac{\ln(\sin x)}{\tan x} \, dx = \int u \, du = \frac{1}{2} u^2 + C = \frac{1}{2} (\ln(\sin x))^2 + C
\]

10. (7.2 # 22)

\[\int_0^\pi x (\sin x + \cos x) \, dx\]

Let \( u = x \) and \( dv = \sin x + \cos x \, dx \). Then \( du = \, dx \) and \( v = -\cos x + \sin x \). We can then rewrite the integral as:

\[
\int_0^\pi x (\sin x + \cos x) \, dx = \int_0^\pi u \, dv = uv \bigg|_0^\pi - \int_0^\pi v \, du = (-x \cos x + x \sin x) \bigg|_0^\pi - \int_0^\pi \sin x - \cos x \, dx = (-x \cos x + x \sin x) \bigg|_0^\pi + (\cos x + \sin x) \bigg|_0^\pi = (-\pi(-1) + 0) - (0) + (-1 + 0) - (1 - 0) = \pi - 2
\]

11. (7.2 # 26)

\[\int e^{2x} \sin(e^x) \, dx\]
First we will do a substitution. Let \( w = e^x \), then \( dw = e^x \, dx \). Then the integral can be rewritten as:

\[
\int e^{2x} \sin(e^x) \, dx = \int w \sin w \, dw
\]

This new integral is like the one in problem 6 (7.2 #2). So as in that problem, we will now use integration by parts. Let \( u = w \) and \( dv = \sin w \, dw \). Then \( du = dw \) and \( v = -\cos w \). So by integration by parts, we have:

\[
\int e^{2x} \sin(e^x) \, dx = \int w \sin w \, dw = -w \cos w + \int \cos w \, dw = -w \cos w + \sin w + C = -e^x \cos(e^x) + \sin(e^x) + C
\]