Math 1352-11 — WW04 Solutions

October 8, 2008

Assigned problems: 7.1 – 2, 4, 8, 12, 28; 7.2 – 2, 4, 8, 16, 22, 26

Always read through the solution sets even if your answer was correct.

1. (7.1 #2)

$$\int \frac{\ln x}{x}\, dx$$

Let $u = \ln x$, then $du = \frac{1}{x}\, dx$, and we rewrite the integral as:

$$\begin{aligned} \int \frac{\ln x}{x}\, dx &= \int u\, du \\ &= \frac{1}{2}u^2 + C \\ &= \boxed{\tfrac{1}{2}(\ln x)^2 + C} \end{aligned}$$

2. (7.1 # 4)

$$\int \cos(x) e^{\sin(x)}\, dx$$

Let $u = \sin x$, then $du = \cos x\, dx$, and we can write the integral as:

$$\begin{aligned} \int \cos(x) e^{\sin(x)}\, dx &= \int e^u\, du \\ &= e^u + C \\ &= \boxed{e^{\sin x} + C} \end{aligned}$$

3. (7.1 # 8)

$$\int \frac{4x^3 - 4x}{x^4 - 2x^2 + 3}\, dx$$

Let $u = x^4 - 2x^2 + 3$, then $du = 4x^3 - 4x\, dx$, and we can write the integral as:

$$\begin{aligned} \int \frac{4x^3 - 4x}{x^4 - 2x^2 + 3}\, dx &= \int \frac{du}{u} \\ &= \ln u + C \\ &= \boxed{\ln(x^4 - 2x^2 + 3) + C} \end{aligned}$$

4. (7.1 # 12)

$$\int \frac{(2x-1)}{(4x^2-4x)^2}\,dx$$

First we'll factor the denominator as $(4x^2-4x)^2 = [4(x^2-x)]^2 = 16(x^2-x)^2$ giving us

$$\frac{1}{16}\int \frac{(2x-1)}{(x^2-x)^2}\,dx$$

Now we let $u = x^2 - x$, then $du = 2x - 1$, and we can write the integral as

$$\begin{aligned} \frac{1}{16}\int \frac{(2x-1)}{(x^2-x)^2}\,dx &= \frac{1}{16}\int \frac{du}{u^2} \\ &= -\frac{1}{16}u^{-1} + C \\ &= \boxed{-\tfrac{1}{16(x^2-x)} + C} \end{aligned}$$

5. (7.1 # 28)

$$\int x\sqrt{x+1}\,dx$$

Let $u = x + 1$, then $du = dx$ (and note that $x = u - 1$). Then we can write the integral as

$$\begin{aligned} \int x\sqrt{x+1}\,dx &= \int (u-1)u^{1/2}\,du \\ &= \int u^{3/2} - u^{1/2}\,du \\ &= \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C \\ &= \boxed{\tfrac{2}{5}(x+1)^{5/2} - \tfrac{2}{3}(x+1)^{3/2} + C} \end{aligned}$$

6. (7.2 # 2)

$$\int x\sin x\,dx$$

Let $u = x$ and $dv = \sin x\,dx$. Then $du = dx$ and $v = -\cos x$. Integration by parts gives us:

$$\begin{aligned} \int x\sin x\,dx &= \int u\,dv \\ &= uv - \int v\,du \\ &= -x\cos x + \int \cos x\,dx \\ &= \boxed{-x\cos x + \sin x + C} \end{aligned}$$

7. (7.2 # 4)

$$\int x \tan^{-1} x \, dx$$

Let $u = \tan^{-1} x$ and $dv = x\,dx$. Then $du = \frac{1}{x^2+1}\,dx$ and $v = \frac{1}{2}x^2$. Integration by parts gives us:

$$\begin{aligned}
\int x \tan^{-1} x \, dx &= \int u \, dv \\
&= uv - \int v \, du \\
&= \frac{1}{2}x^2 \tan^{-1} x - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx \\
&\quad \textit{(Use integration table for this integral. Formula 57, Appendix D.)} \\
&= \frac{1}{2}x^2 \tan^{-1} x - \frac{1}{2}\left(x - \tan^{-1} x\right) + C
\end{aligned}$$

8. (7.2 # 8)

$$\int e^{2x} \sin(3x)\,dx$$

Let $u = e^{2x}$ and $dv = \sin(3x)\,dx$. Then $du = 2e^{2x}\,dx$ and $v = -\frac{1}{3}\cos(3x)$. Integration by parts gives us:

$$\begin{aligned}
\int e^{2x} \sin(3x)\,dx &= \int u \, dv \\
&= uv - \int v \, du \\
&= -\frac{1}{3}e^{2x}\cos(3x) + \frac{2}{3}\int e^{2x}\cos(3x)\,dx
\end{aligned}$$

We still can't solve the new integral, but it looks like if we try integration by parts again, we'll get back the original integral.

Let $u = e^{2x}$ and $dv = \cos(3x)\,dx$. Then $du = 2e^{2x}\,dx$ and $v = \frac{1}{3}\sin(3x)$ and we'll get:

$$\begin{aligned}
\int e^{2x} \sin(3x)\,dx &= -\frac{1}{3}e^{2x}\cos(3x) + \frac{2}{3}\left(\frac{1}{3}e^{2x}\sin(3x) - \frac{2}{3}\int e^{2x}\sin(3x)\,dx\right) \\
&= -\frac{1}{3}e^{2x}\cos(3x) + \frac{2}{9}e^{2x}\sin(3x) - \frac{4}{9}\int e^{2x}\sin(3x)\,dx
\end{aligned}$$

Notice that the new integral on the right ($\int v\,du$) is equal to $-\frac{4}{9}$ times our original integral. We can move this integral to the left-hand side and solve for the value of the integral:

$$\begin{aligned}\left(1+\frac{4}{9}\right)\int e^{2x}\sin(3x)\,dx &= -\frac{1}{3}e^{2x}\cos(3x)+\frac{2}{9}e^{2x}\sin(3x)\\ \frac{13}{9}\int e^{2x}\sin(3x)\,dx &= -\frac{1}{3}e^{2x}\cos(3x)+\frac{2}{9}e^{2x}\sin(3x)\\ \int e^{2x}\sin(3x)\,dx &= \boxed{-\tfrac{3}{13}e^{2x}\cos(3x)+\tfrac{2}{13}e^{2x}\sin(3x)+C}\end{aligned}$$

9. (7.2 # 16)

$$\int \frac{\ln(\sin x)}{\tan x}\,dx$$

There are multiple ways to do this integral, but I found substitution to be easiest. Let $u=\ln(\sin x)$, then using the Chain Rule, $du=\frac{1}{\sin x}\cdot\cos x\,dx=\frac{\cos x}{\sin x}\,dx=\frac{1}{\tan x}\,dx$. So we can rewrite the integral as:

$$\begin{aligned}\int \frac{\ln(\sin x)}{\tan x}\,dx &= \int u\,du\\ &= \frac{1}{2}u^2+C\\ &= \frac{1}{2}\left(\ln(\sin x)\right)^2+C\end{aligned}$$

10. (7.2 # 22)

$$\int_0^\pi x\left(\sin x+\cos x\right)\,dx$$

Let $u=x$ and $dv=\sin x+\cos x\,dx$. Then $du=dx$ and $v=-\cos x+\sin x$. We can then rewrite the integral as:

$$\begin{aligned}\int_0^\pi x\left(\sin x+\cos x\right)\,dx &= \int_0^\pi u\,dv\\ &= uv\,|_0^\pi-\int_0^\pi v\,du\\ &= \left(-x\cos x+x\sin x\right)|_0^\pi-\int_0^\pi \sin x-\cos x\,dx\\ &= \left(-x\cos x+x\sin x\right)|_0^\pi+\left(\cos x+\sin x\right)|_0^\pi\\ &= (-\pi(-1)+0)-(0)+(-1+0)-(1-0)\\ &= \boxed{\pi-2}\end{aligned}$$

11. (7.2 # 26)

$$\int e^{2x}\sin(e^x)\,dx$$

First we will do a substitution. Let $w = e^x$, then $dw = e^x\ dx$. Then the integral can be rewritten as:

$$\int e^{2x} \sin(e^x)\ dx = \int w \sin w\ dw$$

This new integral is like the one in problem 6 (7.2 #2). So as in that problem, we will now use integration by parts. Let $u = w$ and $dv = \sin w\ dw$. Then $du = dw$ and $v = -\cos w$. So by integration by parts, we have:

$$\begin{aligned}
\int e^{2x} \sin(e^x)\ dx &= \int w \sin w\ dw \\
&= -w \cos w + \int \cos w\ dw \\
&= -w \cos w + \sin w + C \\
&= -e^x \cos(e^x) + \sin(e^x) + C
\end{aligned}$$