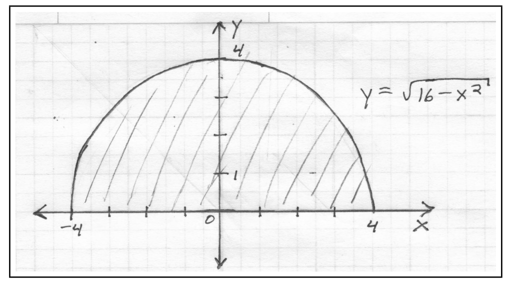
Math 1352-11 — WW02 Solutions

September 10, 2008

Assigned problems: 6.2 – 2, 12, 14, 18, 42, 44; 6.3 – 4, 24, 28

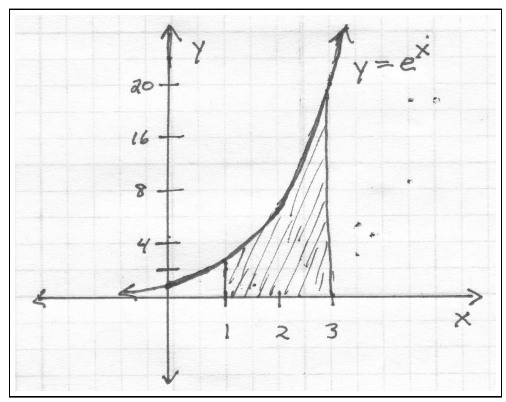
You can see the answers to webwork assignments shortly after the deadline. In these solution sets, I will write out more complete solutions so you can see how I went about solving the problems. *Always read through the solution sets even if your answer on webwork was correct.*

1. (6.2 #2) $y = \sqrt{16 - x^2}$
   Each cross-section is a square perpendicular to the x-axis on the semicircular base. One side of the square has length $\sqrt{16 - x^2}$, so the area of one square slice is $\sqrt{16 - x^2} \times \sqrt{16 - x^2} = 16 - x^2$. (Notice that $16 - x^2 \geq 0$ on $[-4, 4]$.) So the volume is given by



$$\begin{aligned} V &= \int_{-4}^{4} \left(16 - x^2\right)\, dx \\ &= 16x - \frac{1}{3}x^3 \bigg|_{-4}^{4} \\ &= \boxed{256/3} \end{aligned}$$

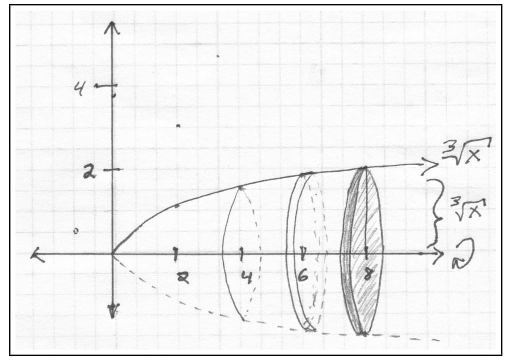
2. (6.2 #12) $y = e^x$
   Each slice is a semicircle perpendicular to the base given by the region between $y = e^x$ and the x-axis on $[1, 3]$. Therefore the radius of each semicircle is $r = \frac{1}{2}e^x$ and each area is $\frac{1}{2}\pi r^2 = \frac{1}{8}\pi e^{2x}$. The whole volume is then



$$\begin{aligned} V &= \frac{\pi}{8} \int_{1}^{3} e^{2x}\, dx \\ &= \frac{\pi}{8} \left(\frac{1}{2}e^{2x}\right)\bigg|_{1}^{3} \\ &= \boxed{\tfrac{\pi}{16}\left(e^6 - e^2\right)} \end{aligned}$$

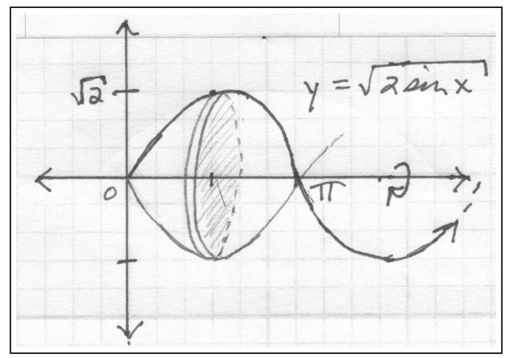
3. (6.2 #14) $y = x^{1/3}$ on $[0, 8]$ revolved about the x-axis.
If we take a slice perpendicular to the x-axis, we have a disk with radius $r = \sqrt[3]{x}$ and area $\pi r^2 = \pi x^{2/3}$.
So the volume of the solid of revolution is

$$\begin{aligned} V &= \int_0^8 \pi x^{2/3}\, dx \\ &= \frac{3\pi}{5} x^{5/3} \Big|_0^8 \\ &= \frac{3\pi}{5}(32 - 0) \\ &= \boxed{\tfrac{96\pi}{5}} \end{aligned}$$

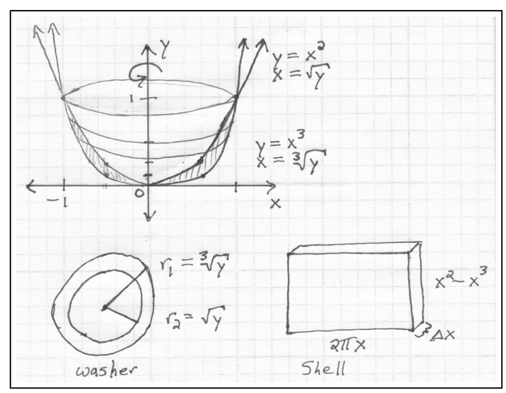


4. (6.2 #18) $y = \sqrt{2 \sin x}$ on $[0, \pi]$ revolved about the x-axis. A vertical slice is a disk with radius $r = \sqrt{2 \sin x}$ and area $\pi r^2 = 2\pi \sin x$. (Notice that $2 \sin x \geq 0$ on $[0, \pi]$). So the volume is given by

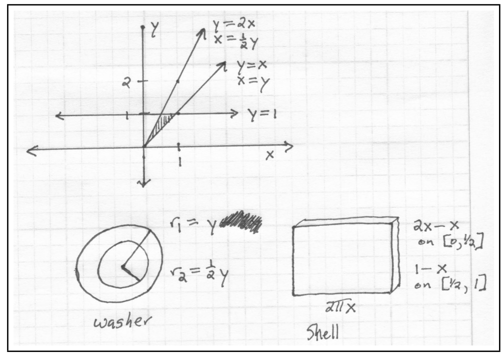
$$\begin{aligned} V &= \int_0^\pi 2\pi \sin x\, dx \\ &= (-2\pi \cos x) \big|_0^\pi \\ &= -2\pi \cos \pi - (-2\pi \cos 0) \\ &= \boxed{4\pi} \end{aligned}$$



5. (6.2 #42) Region bounded by $y = x^2$ and $y = x^3$ revolved about the y-axis.
The two curves intersect when $x^2 = x^3$ giving $x^2(x - 1) = 0$ or $x = 0, 1$.
   (a) Revolving horizontal slices about the y-axis, we get **washers** with outer radius $\sqrt[3]{y}$ and inner radius $\sqrt{x}$, since $x = \sqrt[3]{y}$ leads $x = \sqrt{y}$ on the y interval $[0, 1]$. So the area of a washer is $A = \pi x^{2/3} - \pi x$, and the volume is given by
   $\boxed{V = \pi \int_0^1 y^{2/3} - y\, dy}$
   (b) Revolving vertical slices about the y-axis we get **cylindrical shells**. Notice that on the x interval $[0, 1]$, $x^2 \geq x^3$. So each shell has height $x^2 - x^3$ and radius x, and the volume of one shell at position x is $2\pi x(x^2 - x^3) = 2\pi(x^3 - x^4)$. So the total volume is given by
   $\boxed{V = 2\pi \int_0^1 x^3 - x^4\, dx}$
   (c) Integrating with either (a) or (b) gives
   $\boxed{V = \frac{\pi}{10}}$



6. (6.2 #44) Region bounded by $y = x$, $y = 2x$, and $y = 1$ revolved about the y-axis.



   (a) Revolving horizontal slices about the y-axis, we get **washers** with outer radius y and inner radius $\frac{1}{2}y$, so the area of a washer is $A = \pi(y^2 - \frac{1}{4}y^2) = \frac{3}{4}\pi y^2$. Thus the volume is $\boxed{V = \frac{3}{4}\pi \int_0^1 y^2 \, dy}$

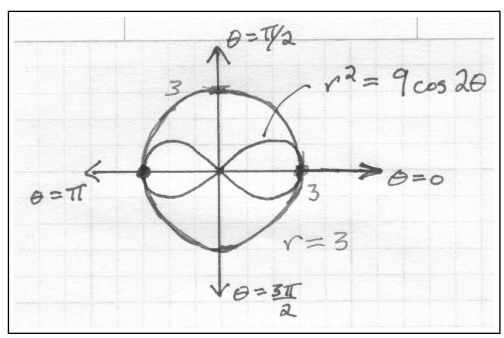
   (b) Revolving vertical slices about the y-axis we get **cylindrical shells**. Notice that on $[0, \frac{1}{2}]$ the height of the shell is $2x - x$, and on $[\frac{1}{2}, 1]$ the height is $1 - x$. So we have two integrals to compute for the volume: $\boxed{V = 2\pi \int_0^{\frac{1}{2}} x^2 \, dx + 2\pi \int_{\frac{1}{2}}^1 x - x^2 \, dx}$

   (c) Integrating with either (a) or (b) gives $\boxed{V = \frac{\pi}{4}}$

7. (6.3 #4) Categorize the curves:

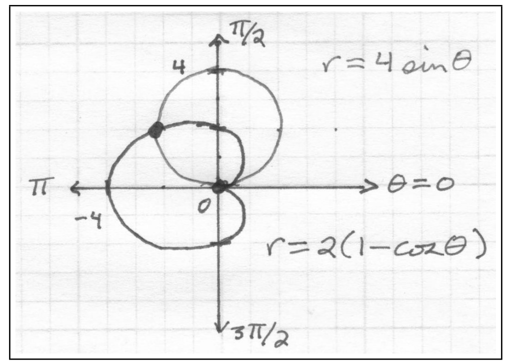
   1. $r = 2\sin(2\theta)$: rose with 4 petals and $\frac{\pi}{4}$ rotation. $\boxed{\text{B}}$.
   2. $r^2 = 2\cos(2\theta)$: 2-loop lemniscate, standard form. $\boxed{\text{C}}$
   3. $r = 5\cos(60°)$: radius is a constant value, so this is a circle. $\boxed{\text{E}}$
   4. $r = 5\sin(8\theta)$: rose with 16 petals, $\frac{\pi}{4}$ rotation. $\boxed{\text{B}}$
   5. $r\theta = 3$: not one of our standard forms. $\boxed{\text{G}}$
   6. $r^2 = 9\cos(2\theta - \frac{\pi}{4})$: lemniscate. $\boxed{\text{C}}$
   7. $r = \sin(3(\theta + \frac{\pi}{6})) = \sin(\theta + 2\theta + \frac{\pi}{2}) = \sin(\theta + \frac{\pi}{2}) = \cos\theta$: rose with 1 circular petal. $\boxed{\text{B}}$
   8. $\cos\theta = 1 - r$: reorder as $r = 1 - \cos\theta$; cardioid. $\boxed{\text{A}}$

8. (6.3 #24) Points of intersection for $r^2 = 9\cos(2\theta)$ (lemniscate with 2 loops of length $\sqrt{9} = 3$) and $r = 3$ (circle of radius 3).
   Graph to determine intersection points.
   Curves intersect at $(r, \theta) = (3, 0)$ and $(3, \pi)$.



9. (6.3 #28) Points of intersection for $r = 2(1-\cos\theta)$ (cardioid) and $r = 4\sin\theta$ (rose with 1 circular petal and $\frac{\pi}{2}$ rotation).
   Graph to see approx. intersections.
   Intersect at pole and one other point.
   Find the other point with:

$$\begin{aligned}
2(1-\cos\theta) &= 4\sin\theta \\
1-\cos\theta &= 2\sin\theta \\
(1-\cos\theta)^2 &= 4\sin^2\theta \quad \text{(square both sides)} \\
1-2\cos\theta+\cos^2\theta &= r\sin^2\theta \;=\; 4(1-\cos^2\theta) \\
5\cos^2\theta-2\cos\theta-3 &= 0 \\
(5\cos\theta+3)(\cos\theta-1) &= 0 \\
\cos\theta &= 1, -\frac{3}{5} \\
\theta &= 0,\ \cos^{-1}\left(-\frac{3}{5}\right)
\end{aligned}$$



$\theta = 0$ gives $r = 0$, which is the Pole.
$\theta = \cos^{-1}(-3/5)$ gives $r = 4\sin(\cos^{-1}(-3/5))$ or
$r = 2(1-\cos(\cos^{-1}(-3/5))) = 2(1-(-3/5)) = 16/5$.
So points of intersection are the Pole and

$\boxed{(\frac{16}{5}, \cos^{-1}(-3/5))}$