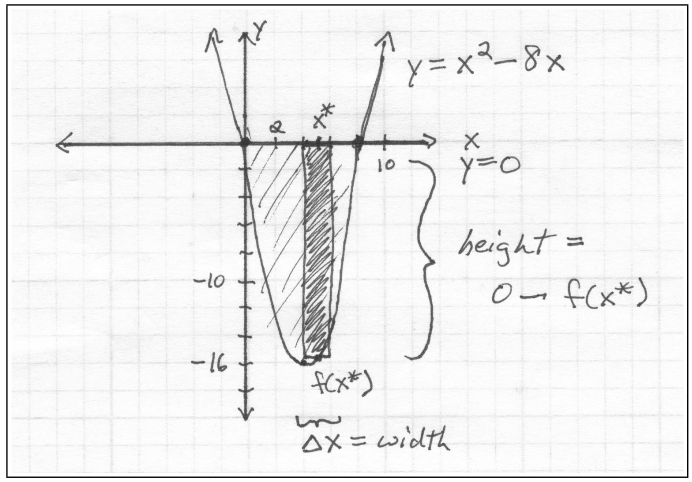
Math 1352-11 — WW01 Solutions

September 3, 2008

Assigned problems: 6.1 – 2, 4, 8, 12, 18, 20

You can see the answers to webwork assignments shortly after the deadline. In these solution sets, I will write out more complete solutions so you can see how I went about solving the problems. *Always read through the solution sets even if your answer on webwork was correct.*

1. (6.1 #2) $y = x^2 - 8x$, $y = 0$
   Curves intersect at $x = 0, 8$.
   $(y = 0) \geq (y = x^2 - 8x)$ on whole interval $[0, 8]$
   Using vertical strips:

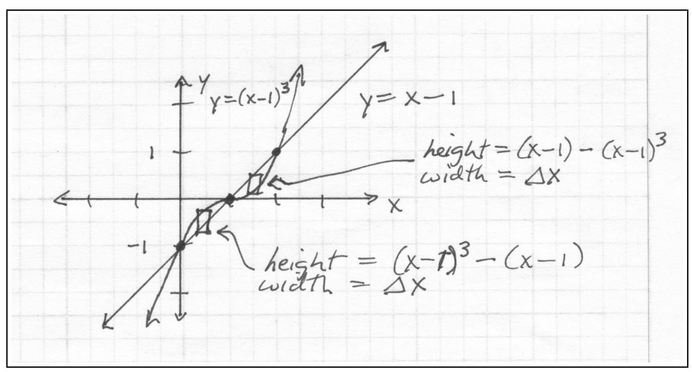


$$\begin{aligned} A &= \int_0^8 0 - (x^2 - 8x)\, dx \\ &= \int_0^8 -x^2 + 8x\, dx \\ &= -\frac{1}{3}x^3 + 4x^2 \Big|_0^8 \\ &= -\frac{1}{3}(8)^3 + 4(8)^2 - 0 \\ &= \boxed{256/3} \end{aligned}$$

2. (6.1 #4) $y = (x-1)^3$, $y = x - 1$
   Curves intersect at:

$$\begin{aligned} (x-1)^3 - (x-1) &= 0 \\ x^3 - 3x^2 + 2x &= 0 \\ x &= 0, 1, 2 \end{aligned}$$

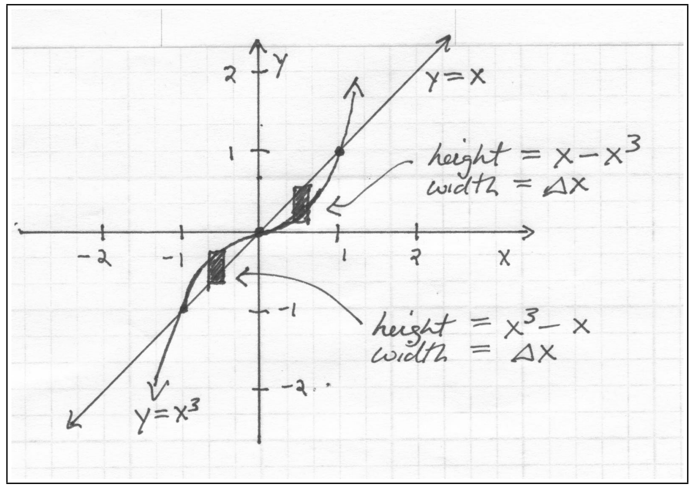
   $(x-1)^3 \geq (x-1)$ on $[0, 1]$
   $(x-1) \geq (x-1)^3$ on $[1, 2]$
   Using vertical strips:



$$\begin{aligned} A &= \int_0^1 (x-1)^3 - (x-1)\, dx + \int_1^2 (x-1) - (x-1)^3\, dx \\ &= \int_0^1 x^3 - 3x^2 + 2x\, dx + \int_1^2 -x^3 + 3x^2 - 2x\, dx \\ &= \frac{1}{4}x^4 - x^3 + x^2 \Big|_0^1 + -\frac{1}{4}x^4 + x^3 - x^2 \Big|_1^2 \\ &= \boxed{1/2} \quad \text{(Note that we could also have done 2 times one of these integrals by symmetry.)} \end{aligned}$$

3. (6.1 #8) $y = x^3$, $y = x$, on $[-1, 1]$
   Curves intersect at $x^3 = x$ giving $x = -1, 0, 1$.
   $x^3 \leq x$ on $[0, 1]$
   $x^3 \geq x$ on $[-1, 0]$
   Using vertical strips:

$$\begin{aligned} A &= \int_{-1}^{1} x^3 - x \, dx + \int_{0}^{1} x - x^3 \, dx \\ &= \frac{1}{4}x^4 - \frac{1}{2}x^2 \Big|_{-1}^{0} + \frac{1}{2}x^2 - \frac{1}{4}x^4 \Big|_{0}^{1} \\ &= \left(-\frac{1}{4} + \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) \\ &= \boxed{\frac{1}{2}} \end{aligned}$$

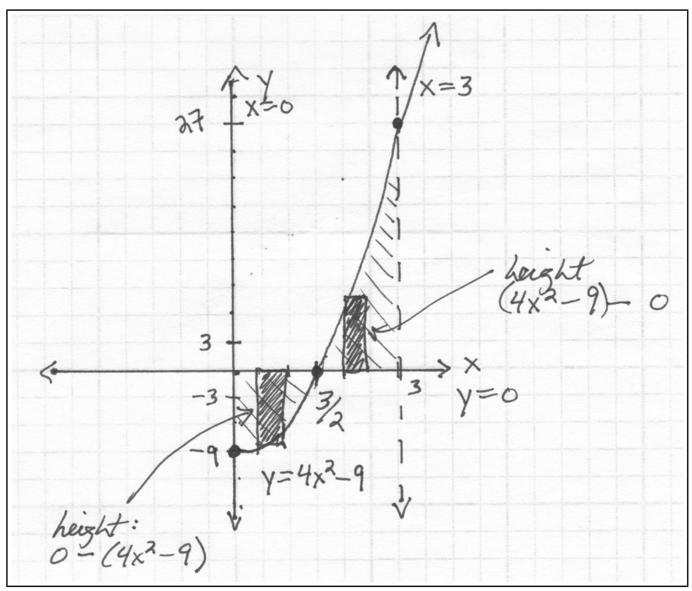


   We could also have used symmetry here giving:

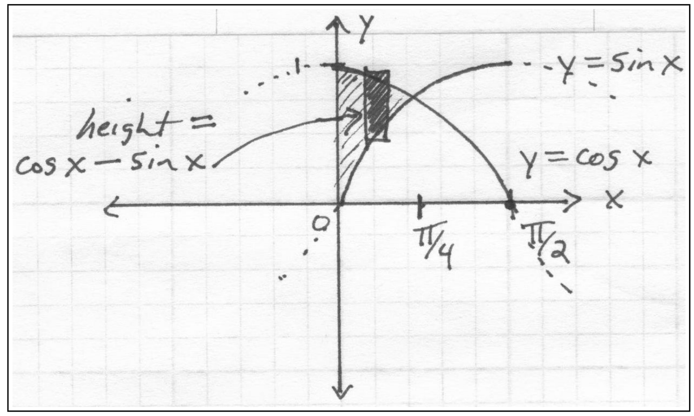
$$\begin{aligned} A &= 2\int_{-1}^{1} x^3 - x \, dx \\ &= 2\int_{0}^{1} x - x^3 \, dx = \boxed{\frac{1}{2}} \end{aligned}$$

4. (6.1 #12) $y = 4x^2 - 9$, $y = 0$, $x = 0$, $x = 3$
   Curves $y = 4x^2 - 9$ and $y = 0$ intersect on the x interval $[0, 3]$ at $x = 3/2$.
   $0 \geq 4x^2 - 9$ on $[0, 3/2]$
   $4x^2 - 9 \geq 0$ on $[3/2, 3]$
   Using vertical strips:

$$\begin{aligned} A &= \int_{0}^{\frac{3}{2}} 0 - (4x^2 - 9) \, dx + \int_{\frac{3}{2}}^{3} (4x^2 - 9) - 0 \, dx \\ &= -\frac{4}{3}x^3 + 9x \Big|_{0}^{\frac{3}{2}} + \frac{4}{3}x^3 - 9x \Big|_{\frac{3}{2}}^{3} \\ &= 9 + 18 = \boxed{27} \end{aligned}$$

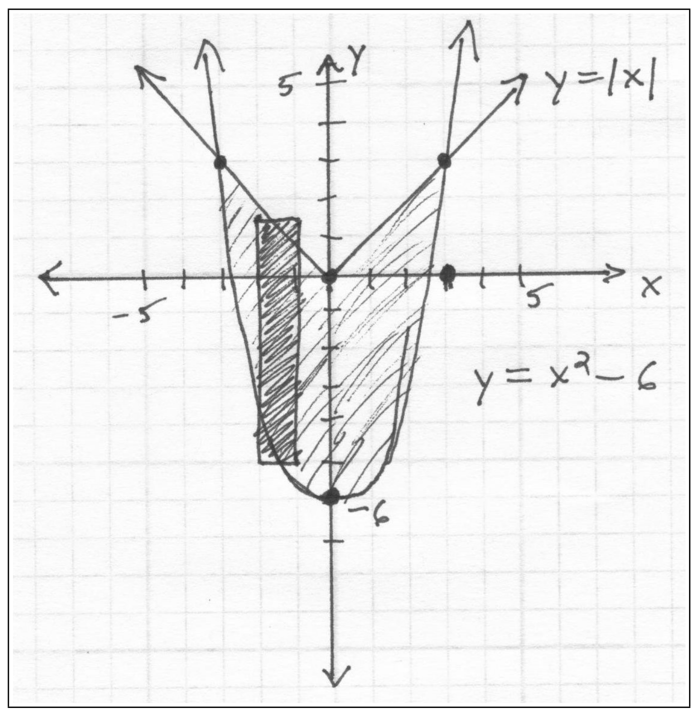


5. (6.1 #18)
$y = \sin x$, $y = \cos x$, on $[0, \pi/4]$
$\cos x \geq \sin x$ on $[0, \pi/4]$
Using vertical strips:



$$\begin{aligned} A &= \int_0^{\frac{\pi}{4}} \cos x - \sin x \, dx \\ &= \sin x + \cos x \Big|_0^{\frac{\pi}{4}} \\ &= 2\left(\frac{\sqrt{2}}{2}\right) - (0 - 1) \\ &= \boxed{\sqrt{2} + 1} \end{aligned}$$

6. (6.1 #20)
$y = |x|$, $y = x^2 - 6$, on $[-3, 3]$
$|x| \geq x^2 - 6$ on $[-3, 3]$, but note that we have to break $|x|$ into the two intervals $[-3, 0]$ and $[0, 3]$.
Using vertical strips:



$$\begin{aligned} A &= \int_{-3}^{0} -x - x^2 + 6 \, dx \; + \int_0^3 x - x^2 + 6 \, dx \\ &= -\frac{1}{3}x^3 - \frac{1}{2}x^2 + 6x \Big|_{-3}^{0} + -\frac{1}{3}x^3 + \frac{1}{2}x^2 + 6x \Big|_0^3 \\ &= 0 - \left(9 - \frac{9}{2} - 18\right) + \left(-9 + \frac{9}{2} + 18\right) - 0 \\ &= \boxed{27} \end{aligned}$$

Or again, by symmetry, we could get

$$\begin{aligned} A &= 2\int_0^3 x - x^2 + 6 \, dx \\ &= \boxed{27} \end{aligned}$$