Math 1352-11 — HW03 Solutions
October 9, 2008

Assigned problems: 7.1 – 18, 24, 52, 56; 7.2 – 18, 38, 52

Always read through the solution sets even if your answer was correct.

- (7.1 #18)

  $$\int \frac{dx}{a+be^{2x}}$$

  We can solve this integral using the text integral table Formula 489 with $p = a$, $q = b$, $u = x$, and $a = 2$: .

  $$\begin{aligned} \int \frac{dx}{a+be^{2x}} &= \frac{u}{p} - \frac{1}{ap}\ln|p+qe^{au}| + C \\ &= \boxed{\frac{x}{a} - \frac{1}{2a}\ln|a+be^{2x}| + C} \end{aligned}$$

- (7.1 #24)

  $$\int x\sin^{-1}\ dx$$

  We can solve this integral using the text integral table Formula 452 with $u = x$ and $a = 1$:

  $$\begin{aligned} \int x\sin^{-1}\ dx &= \left(\frac{u^2}{2} - \frac{a^2}{4}\right)\sin^{-1}\frac{u}{a} + \frac{u\sqrt{a^2-u^2}}{4} + C \\ &= \boxed{\left(\frac{x^2}{2} - \frac{1}{4}\right)\sin^{-1}x + \frac{x\sqrt{1-x^2}}{4} + C} \end{aligned}$$

- (7.1 #52) Area of region bounded by $y = \frac{2x}{\sqrt{x^2+9}}$, $y = 0$, $x = 0$, and $x = 4$.

  y is positive, so the area is:

  $$\begin{aligned} A &= \int_0^4 \frac{2x}{\sqrt{x^2+9}}\,dx \\ &= \text{We'll use the text integral table Formula 173 with } u = x \text{ and } a^2 = 9 \\ &= 2\sqrt{u^2+a^2}\Big|_{x=0}^{x=4} \\ &= 2\sqrt{x^2+9}\Big|_0^4 \\ &= 2\sqrt{25} - 2\sqrt{9} \\ &= \boxed{4} \end{aligned}$$

  Alternatively, we could have used substitution with $u = x^2 + 9$ and $du = 2x\ dx$.

- (7.1 #56)

$$\begin{aligned}
V &= \pi \int_1^2 \sqrt{4-y^2}\, dy \\
&\quad \text{(Text integral table Formula 231 with } a=2 \text{ and } u=y\text{)} \\
&= \pi \left(\frac{u\sqrt{a^2-u^2}}{2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} \right)\Bigg|_{y=1}^{y=2} \\
&= \pi \left(\frac{y\sqrt{4-y^2}}{2} + 2\sin^{-1}\frac{y}{2} \right)\Bigg|_1^2 \\
&= \pi \left(0 + 2\sin^{-1}(1) - \frac{\sqrt{3}}{2} - 2\sin^{-1}\left(\frac{1}{2}\right) \right) \\
&= \pi \left(\pi - \frac{\sqrt{3}}{2} - \frac{\pi}{6} \right) \\
&= \boxed{\pi^2 - \frac{\sqrt{3}\pi}{2} - \frac{\pi^2}{6} \approx 3.8590}
\end{aligned}$$

- (7.2 #18)

$$\int_1^e x^3 \ln x\, dx$$

We'll integrate by parts using $u = \ln x$ and $dv = x^3\, dx$. Then $du = \frac{1}{x}\, dx$ and $v = \frac{1}{4}x^4$, giving us:

$$\begin{aligned}
\int_1^e x^3 \ln x\, dx &= \int_{x=1}^{x=e} u\, dv \\
&= uv\big|_{x=1}^{x=e} - \int_{x=1}^{x=e} v\, du \\
&= \frac{1}{4}x^4 \ln x\Big|_1^e - \frac{1}{4}\int_1^e \frac{1}{x}x^4\, dx \\
&= \frac{1}{4}x^4 \ln x\Big|_1^e - \frac{1}{4}\int_1^e x^3\, dx \\
&= \frac{1}{4}x^4 \ln x\Big|_1^e - \frac{1}{16}x^4\Big|_1^e \\
&= \frac{1}{4}\left(e^4\ln(e) - 1\ln(1)\right) - \frac{1}{16}\left(e^4 - 1\right) \\
&= \boxed{\tfrac{1}{16}\left(3e^4 + 1\right)}
\end{aligned}$$

- (7.2 #38) Let R be the region under the curve $y = \sin x + \cos x$ on the interval $[0, \pi/4]$. We want to find the volume generated by revolving R about the y-axis.

  We'll use the method of cylindrical shells. (To use Disks in this problem, we would have to integrate in the y direction, so we would have to rewrite $y = \sin x + \cos x$ to give x as a function of y.) The height of a shell would be $y = (\sin x + \cos x) - 0$ and it's radius is x. Therefore the total volume is:

$$
\begin{aligned}
V &= 2\pi \int_0^{\pi/4} x\,(\sin x + \cos x)^2 \; dx \\
&= 2\pi \int_0^{\pi/4} x \sin x \; dx \; + \; 2\pi \int_0^{\pi/4} x \cos x \; dx
\end{aligned}
$$

(We'll use integration by parts on both integrals. In the first, let $u = x$ and $dv = \sin x\ dx$ so that $du = dx$ and $v = -\cos x$. In the second, let $u = x$ and $dv = \cos x\ dx$, so that $du = dx$ and $v = \sin x$)

$$
\begin{aligned}
&= 2\pi \left(-x \cos x \Big|_0^{\pi/4} + \int_0^{\pi/4} \cos x \; dx + \; x \sin x \Big|_0^{\pi/4} - \int_0^{\pi/4} \sin x \; dx \right) \\
&= 2\pi \left(-x \cos x + \sin x + x \sin x + \cos x \right) \Big|_0^{\pi/4} \\
&= 2\pi \left(\cancel{-\frac{\pi}{4} \cdot \frac{\sqrt{2}}{2}} + \frac{\sqrt{2}}{2} + \cancel{\frac{\pi}{4} \cdot \frac{\sqrt{2}}{2}} + \frac{\sqrt{2}}{2} \right) - 2\pi \left(0 + 0 + 0 + 1 \right) \\
&= \boxed{2\pi(\sqrt{2} - 1)}
\end{aligned}
$$

- (7.2 #52)

$$
\int \frac{(2x-1)}{x^2} e^{2x} \; dx \;=\; 2 \int x^{-1} e^{2x} \; dx \; - \; \int x^{-2} e^{2x} \; dx
$$

  We'll start with integration by parts on the first of the two integrals. Let $u = x^{-1}$ and $dv = e^{2x}\ dx$. Then $du = -x^{-2}\ dx$ and $v = \frac{1}{2} e^{2x}$.

$$
\begin{aligned}
\int \frac{(2x-1)}{x^2} e^{2x} \; dx &= 2 \left(\frac{1}{2} x^{-1} e^{2x} + \frac{1}{2} \int x^{-2} e^{2x} \; dx \right) \; - \; \int x^{-2} e^{2x} \; dx \\
&= x^{-1} e^{2x} \; + \; \int x^{-2} e^{2x} \; dx \; - \; \int x^{-2} e^{2x} \; dx \\
&= x^{-1} e^{2x} \; + \; \cancel{\int x^{-2} e^{2x} \; dx} \; - \; \cancel{\int x^{-2} e^{2x} \; dx} \\
&= \boxed{x^{-1} e^{2x} + C}
\end{aligned}
$$

(When I gave a hint about this problem in class, I started with a simple u substitution to simplify it a bit. That is valid, but not really necessary, so I skipped that step here.)