

Math 1352-11 — HW01 Solutions

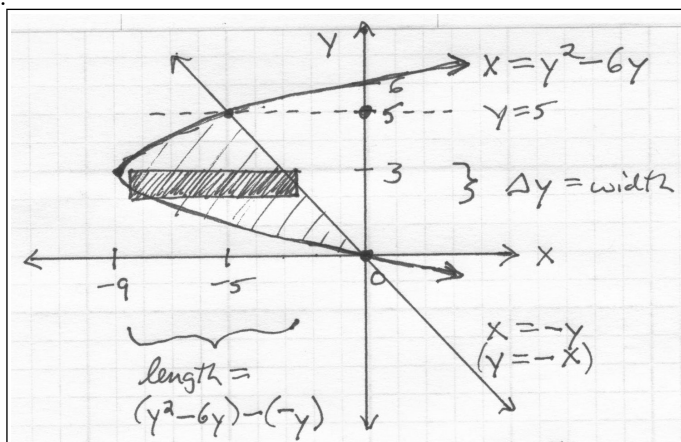
September 4, 2008

Assigned problems: 6.1 – 6, 24, 28

Always read through the solution sets even if your answer was correct.

- (6.1 #6) $x = y^2 - 6y$, $x = -y$
Curves intersect when $y^2 - 6y - (-y) = 0$, i.e., at $y = 0, 5$.
 $x = -y$ leads $x = y^2 - 6y$ on y interval $[0, 5]$.
Using horizontal strips:

$$\begin{aligned} A &= \int_0^5 -y - y^2 + 6y \, dy \\ &= \int_0^5 -y^2 + 5y \, dy \\ &= -\frac{1}{3}y^3 + \frac{5}{2}y^2 \Big|_0^5 \\ &= -\frac{1}{3}(5)^3 + \frac{5}{2}(25) - 0 \\ &= \boxed{125/6} \end{aligned}$$



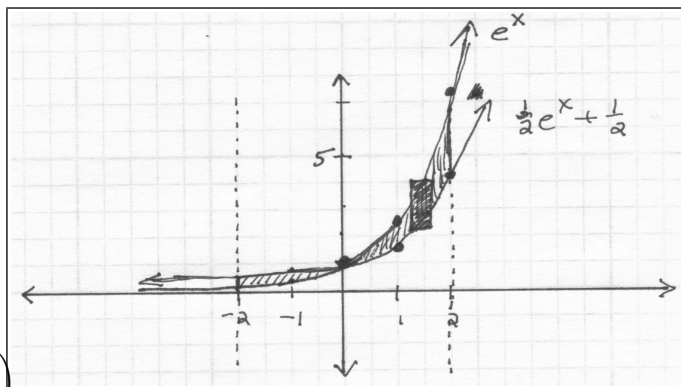
- (6.1 #24) $y = e^x$, $y = \frac{1}{2}e^x + \frac{1}{2}$, on $[-2, 2]$
Curves intersect at $e^x - \frac{1}{2}e^x - \frac{1}{2} = 0$, giving $e^x - 1 = 0$
or $\ln(e^x) = \ln(1)$ giving $x = 0$.

$$e^x \leq \frac{1}{2}e^x + \frac{1}{2} \text{ on } [-2, 0]$$

$$e^x \geq \frac{1}{2}e^x + \frac{1}{2} \text{ on } [0, 2]$$

Using vertical strips:

$$\begin{aligned} A &= \int_{-2}^0 \frac{1}{2}e^x + \frac{1}{2} \, dx + \int_0^2 \frac{1}{2}e^x - \frac{1}{2} \, dx \\ &= \left. \frac{1}{2}e^x + \frac{1}{2}x \right|_{-2}^0 + \left. \frac{1}{2}e^x - \frac{1}{2}x \right|_0^2 \\ &= \left(-\frac{1}{2} \right) - \left(-\frac{1}{2}e^{-2} - 1 \right) + \left(\frac{1}{2}e^2 - 1 \right) - \left(\frac{1}{2} \right) \\ &= \boxed{\frac{1}{2e^2} + \frac{1}{2}e^2 - 1} \end{aligned}$$



- (6.1 #28) $2y = 11 - x$, $y = 7x + 13$, $y = x^2 - 5$

Curves $y = x^2 - 5$ and $2y = 11 - x$ intersect at $x = 3, -\frac{7}{2}$.

$2y = 11 - x$ and $7x + 13$ intersect at $x = -1$;

$y = x^2 - 5$ and $y = 7x + 13$ intersect at $x = -2, 9$.

Using vertical strips, we have:

$$y = 7x + 13 \geq x^2 - 5 \text{ on } [-2, -1]$$

$$2y = 11 - x \geq x^2 - 5 \text{ on } [-1, 3]$$

$$\begin{aligned} A &= \int_{-2}^{-1} 7x + 13 - x^2 + 5 \, dx + \int_{-1}^3 \left(\frac{11}{2} - \frac{x}{2}\right) - x^2 + 5 \, dx \\ &= \left[-\frac{1}{3}x^3 + \frac{7}{2}x^2 + 18x\right]_{-2}^{-1} - \left[\frac{1}{3}x^3 - \frac{1}{4}x^2 + \frac{21}{2}x\right]_{-1}^3 \\ &= \left(\frac{1}{3} + \frac{7}{2} - 18\right) - \left(\frac{8}{3} + 14 - 36\right) + \left(-9 - \frac{9}{4} + \frac{63}{2}\right) - \left(\frac{1}{3} - \frac{1}{4} - \frac{21}{2}\right) \\ &= \boxed{215/6} \end{aligned}$$

