

REVIEW FOR THE FINAL EXAMINATION FALL 2007

1. Evaluate the limits

$$\lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{x^2 - 3x + 2}$$
$$\lim_{x \rightarrow 2} \frac{\sqrt{x} - 2}{x - 4}.$$

2. Using l'Hôpital's theorem, compute the limits

$$\lim_{x \rightarrow \infty} \frac{\ln(\ln x)}{x}$$
$$\lim_{x \rightarrow 0} (\sin x)(\ln x).$$

3. Find the constants a and b so that the function

$$f(x) = \begin{cases} \frac{ax-4}{x-2} & \text{if } x \neq 2 \\ b & \text{if } x = 2 \end{cases}$$

is continuous for all x .

4. Using the definition of the derivative (difference quotient), find the derivative of $f(x) = x^3 + 2x$.
5. Using the rules of differentiation, find the derivative of

$$g(x) = \sqrt{x} \cos x + x \cot x$$
$$y(t) = \frac{x^2 + \tan x}{3x + 2 \tan x}$$
$$h(x) = e^x \tan^{-1} x.$$

6. Using the rules of differentiation, find the derivatives of

$$f(x) = \sin(3x^2 + 2x)$$
$$g(t) = e^{-3t^2} \cos 2t + (2t + 1)^{10}.$$

7. Find the equation of the tangent line to the graph of the function $f(x) = x^2 + 3x + \sin x$ at $x = 0$.
8. Find $\frac{dy}{dx}$ by implicit differentiation if $e^{xy} + y^2 = x^2 + x + 1$.
9. If $x^2 + y^2 = 100$ and $\frac{dx}{dt} = 10$, find $\frac{dy}{dt}$ when $y = 6$.

10. A ball is dropped from a height of 160 ft. A light is located at the same level, 10 ft away from the initial position of the ball. How fast is the ball's shadow moving along the ground one second after the ball is dropped?
11. Find the absolute extrema of the function defined by the equation $f(x) = x^3 - 3x^2$ on the closed interval $[-1, 3]$.
12. Sketch the graph of

$$f(x) = \frac{1}{3}x^3 - 9x.$$

13. Sketch the graph of

$$f(x) = \frac{2x + 3}{x - 1}$$

14. A person plans to fence off a rectangular garden whose one side is the wall of the house. What is the maximal amount of area that can be enclosed given that the length of the fence cannot exceed 100 ft.
15. Find the antiderivatives of the functions

$$f(x) = (x^2 + 1)^2$$
$$g(x) = 3 \cos x + \frac{2}{\sqrt{1 - x^2}}.$$

16. Estimate (using the right endpoints) the integral

$$\int_0^1 (x^3 + 3x) dx$$

by using a Riemann sum $S_n = \sum_{k=1}^n f(k\Delta x)\Delta x$ for $n = 4$.

17. Evaluate the integrals

$$\int_0^1 (2x + 1)^4 dx$$
$$\int \sqrt{3x + 2} dx$$
$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$
$$\int \frac{\ln x}{x} dx.$$

18. Find the area of the subgraph of the parabola $y = x^2 + x + 1$ that lies over the interval $[-2, 2]$.