Announcements

- Homework 12 due today
- Bonus homework due Wednesday, 12/5
- Homework 13 due Wednesday, 12/5
- **Final exam: Monday, 12/10, 10:00 to 12:00, BA 057.**
  - Bluebooks will be provided
  - I'll have practice problems as usual (Monday).
  - You can also buy old finals (and solutions) from the math club. See fliers in math building.
  - (Optional) review session Friday, 12/7, at 1:00 in CH 107
- Extra office hours next week:
  - Monday: 10:00 to 11:30
  - Tuesday: 10:00 to 11:30
  - Wednesday: 10:00 to 11:30
  - Thursday: 10:00 to 11:30
  - Friday: review at 1:00

Area as an integral: Suppose f is continuous and $f(x) \geq 0$ on the closed interval $[a, b]$. Then the area under the curve $y = f(x)$ on $[a, b]$ is given by the definite integral of f on $[a, b]$. I.e.,

$$\text{AREA} = \int_a^b f(x)\, dx$$

If g is cont. and $g \leq 0$ on the closed interval, then $-g(x) \geq 0$ and

$$\text{AREA} = -\int_a^b g(x)\, dx$$

If f is cont. and sometimes positive and sometimes negative, then

$$\int_a^b f(x)\, dx = A_1 - A_2$$

- A_1 is the sum of all areas of regions above the x-axis and below the graph of f (i.e., where $f(x) \geq 0$) and
- A_2 is the sum of all areas of the regions below the x-axis and above the graph of f (i.e., where $f(x) \leq 0$).

Note that the variable inside the definite integral is a "dummy variable", e.g.,

$$\int_{-2}^{1} 4x \, dx = \int_{-2}^{1} 4t \, dt = \int_{-2}^{1} 4u \, du$$

etc.

1st Fundamental Theorem of Calculus

If f is continuous on the interval $[a, b]$ and F is any function that satisfies $F'(x) = f(x)$ throughout the interval, then

$$\int_a^b f(x)\, dx = F(b) - F(a) = F(x)\big|_a^b$$

That is, to compute the definite integral $\int_a^b f(x)\, dx$,

- Find an antiderivative of $f(x)$ on the interval $[a, b]$
- Evaluate it at the limits of integration a and b

2nd Fundamental Theorem of Calculus

Let $f(t)$ be continuous on the interval $[a, b]$, and define the function G by the integral equation

$$G(x) = \int_a^x f(t)\, dt$$

for $a \leq x \leq b$. Then G is an antiderivative of f on $[a, b]$.

That is,

$$G'(x) = \frac{d}{dx}\left[\int_a^x f(t)\, dt\right] = f(x)$$

$$\int_a^b f(x)\,dx \;=\; \text{a number}$$

(for fixed numerical values of a and b)

$$\int_a^x f(t)\,dt \;=\; \text{a function of x}$$

$$\int f(x)\,dx \;=\; \text{a family of functions}$$

Integration by substitution

This is like the Chain Rule backwards.

Let f, g, and u be differentiable functions of x such that

$$f(x) = g(u)\frac{du}{dx}$$

Then

$$\int f(x)\ dx = \int g(u)\frac{du}{dx}\ dx = \int g(u)\ du = G(u) + C$$

where G is an antiderivative of g.