Announcements

- Homework 10 due today.
- Homework 11 due Friday, 11/16/2007.
- **Exam 3 will be Monday 11/19/2007**.

Area as an antiderivative

Theorem: If f is a continous function such that $f(x) \geq 0$ for all x on the closed interval $[a, b]$, then the area bounded by the curve $y = f(x)$, the x-axis, and the vertical lines $x = a$, $x = t$, viewed as a function of t, is an antiderivative of $f(t)$ on $[a, b]$.

Area as the limit of a sum

Suppose f is continuous and $f(x) \geq 0$ throughout the interval $[a, b]$. Then the **area** of the region under the curve $y = f(x)$ over the interval is

$$A = \lim_{\Delta x \to 0} [f(a + \Delta x) + f(a + 2\Delta x) + \cdots + f(a + n\Delta x)]\Delta x,$$

where $\Delta x = \frac{b-a}{n}$.

Summation notation

$$a_1 + a_2 + \cdots + a_n = \sum_{k=1}^{n} a_k$$

Also called **sigma notation**.

We can rewrite the area formula in summation notation:
Let

$$\begin{aligned} S_n &= [f(a+\Delta x) + f(a+2\Delta x) + \cdots + f(a+n\Delta x)]\Delta x \\ &= \sum_{k=1}^{n} f(a+k\Delta x)\Delta x \end{aligned}$$

where $\Delta x = \frac{b-a}{n}$.

Then,

$$\begin{aligned} A &= \lim_{n\to+\infty} S_n \\ &= \lim_{\Delta x\to 0} \sum_{k=1}^{n} f(a+k\Delta x)\Delta x \end{aligned}$$

Basic rules for sums

Constant term rule: $\sum_{k=1}^{n} c = c + c + \cdots + c = nc$

Sum rule: $\sum_{k=1}^{n}(a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$

Scalar mult. rule: $\sum_{k=1}^{n} ca_k = c\sum_{k=1}^{n} a_k$

Linearity rule: $\sum_{k=1}^{n}(ca_k + db_k) = c\sum_{k=1}^{n} a_k + d\sum_{k=1}^{n} b_k$

Subtotal rule: If $1 < m < n$, then

$\sum_{k=1}^{n} a_k = \sum_{k=1}^{m} a_k + \sum_{k=m+1}^{n} a_k$,

Dominance rule: If $a_k \leq b_k$ for $k = 1, 2, \ldots n$, then

$\sum_{k=1}^{n} a_k \leq \sum_{k=1}^{n} b_k$

Summation formulas

$$\sum_{k=1}^{n} 1 = n$$

$$\sum_{k=1}^{n} k = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1}{6}$$

$$\sum_{k=1}^{n} k^3 = 1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$