Announcements

- Homework 10 due Monday 11/12/2007.
- Exam 3 will be **Monday 11/19/2007**.

Integration

- A function F is called an **antiderivative** of a given function f on an interval I if

$$F'(x) = f(x)$$

for all x in I.

- **Theorem: Antiderivatives of the same function differ by a constant.** If F is an antiderivative of the continuous function f, then any other antiderivative , G, of f must have the form

$$G(x) = F(x) + C$$

Antiderivative Notation

The notation

$$\int f(x)\,dx = F(x) + C$$

where C is an arbitrary constant, means that F is an antiderivative of f. It is called the **indefinite integral of f** and satisfies the condition that $F'(x) = f(x)$ for all x in the domain of f.

- Note that $\int f(x)\,dx$ represents a whole family of functions, not just a single solution.

Basic Integration Rules

Constant multiple	$\int cf(u)\,du = c\int f(u)\,du$
Sum	$\int [f(u)+g(u)]\,du = \int f(u)\,du + \int g(u)\,du$
Difference	$\int [f(u)-g(u)]\,du = \int f(u)\,du - \int g(u)\,du$
Linearity	$\int [af(u)+bg(u)]\,du = a\int f(u)\,du + b\int g(u)\,du$
Constant	$\int 0\,du = 0 + C$
Exponential	$\int e^u\,du = e^u + C$
Log & Power	$\int u^n\,du = \begin{cases} \frac{u^{n+1}}{n+1} + C; & n \neq -1 \\ \ln\lvert u\rvert + C; & n = -1 \end{cases}$

Basic Integration Rules - Trig

$$\int \sin u \, du = -\cos u + C$$

$$\int \cos u \, du = \sin u + C$$

$$\int \sec^2 u \, du = \tan u + C$$

$$\int \csc^2 u \, du = -\cot u + C$$

$$\int \sec u \tan u \, du = \sec u + C$$

$$\int \csc u \cot u \, du = -\csc u + C$$

Basic Integration Rules - Inverse Trig

$$\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C$$

$$\int \frac{du}{1+u^2} = \tan^{-1} u + C$$

$$\int \frac{du}{|u|\sqrt{u^2-1}} = \sec^{-1} u + C$$

Area as an Antiderivative

Theorem: If f is a continous function such that $f(x) \geq 0$ for all x on the closed interval $[a, b]$, then the area bounded by the curve $y = f(x)$, the x-axis, and the vertical lines $x = a$, $x = t$, viewed as a function of t, is an antiderivative of $f(t)$ on $[a, b]$.