

## Announcements

- Homework 9 due today.
- Office hours moved this week.
  - Wednesday 11/7 office hours moved to Friday 11/9/2007.
- Homework 10 will be due Monday 11/12/2007.
- Exam 3 will be **Monday 11/19/2007**.

## L'Hôpital's Rule

Let  $f$  and  $g$  be differentiable functions with  $g'(x) \neq 0$  on an open interval containing  $c$  (except possibly  $c$  itself). Suppose

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)}$$

produces an indeterminate form  $\frac{0}{0}$  or  $\frac{\infty}{\infty}$ , and that

$$\lim_{x \rightarrow c} \frac{f'(x)}{g'(x)} = L,$$

where  $L$  is either a finite number,  $+\infty$ , or  $-\infty$ . Then

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = L$$

- L'Hôpital's Rule also applies to one-sided limits and to limits to infinity (i.e., where  $x$  goes to  $+\infty$  or  $-\infty$ ).
- Be careful to check that the limit gives an indeterminate form before applying L'Hôpital's Rule. **Applying it blindly can give you the wrong answer.**

- Note that L'Hôpital's Rule applies only to the indeterminate forms  $\frac{0}{0}$  and  $\frac{\infty}{\infty}$ .
- For other indeterminate forms such as  $1^\infty$ ,  $0^0$ ,  $\infty^0$ ,  $\infty - \infty$ , and  $0\infty$ , need to manipulate into the form  $\frac{0}{0}$  or  $\frac{\infty}{\infty}$ , then evaluate with L'Hôpital's Rule.

### Special limits with $e^x$ and $\ln x$

If  $k$  and  $n$  are positive numbers, then

$$\lim_{x \rightarrow 0^+} \frac{\ln x}{x^n} = -\infty \quad \lim_{x \rightarrow +\infty} \frac{\ln x}{x^n} = 0$$

$$\lim_{x \rightarrow +\infty} \frac{e^{kx}}{x^n} = +\infty \quad \lim_{x \rightarrow +\infty} x^n e^{-kn} = 0$$