Announcements

- Homework 9 due this Monday 11/5/2007.
- Exam 3 will be **Monday 11/19/2007**.

Limits to Infinity

$$\lim_{x \to +\infty} f(x) = L$$

means that $f(x)$ approaches the number L as x increases without bound.

$$\lim_{x \to -\infty} f(x) = M$$

means that $f(x)$ approaches the number M as x decreases without bound.

Infinite Limits

$$\lim_{x \to c} f(x) = +\infty$$

means that f increases without bound as x approaches c (from either side).

$$\lim_{x \to c} g(x) = -\infty$$

means that g decreases without bound as x approaches c (from either side).

Asymptotes

The line $x = c$ is a **vertical asymptote** of the graph of f if either of the one-sided limits

$$\lim_{x \to c^-} f(x) \qquad \text{or} \qquad \lim_{x \to c^+} f(x)$$

is infinite ($+\infty$ or $-\infty$).

The line $y = L$ is a **horizontal asymptote** of the graph of f if

$$\lim_{x \to +\infty} f(x) = L \qquad \text{or} \qquad \lim_{x \to -\infty} f(x) = L$$

Vertical tangents and cusps

Suppose the function f is continuous at the point $P(c, f(c))$. Then the graph of f has

- a **vertical tangent** at P if $\lim_{x\to c^-} f'(x)$ and $\lim_{x\to c^+} f'(x)$ are either both $+\infty$ or both $-\infty$.
- a **cusp** at P if $\lim_{x\to c^-} f'(x)$ and $\lim_{x\to c^+} f'(x)$ are both infinite with opposite signs (one $+\infty$ and the other $-\infty$).

Summary of Graphing Strategy

- Simplify the equation as much as possible
- Find 1st derivatives and critical numbers
- Determine intervals of increase and decrease
- Apply 2nd derivative test
- Determine concavity and points of inflection
- Apply 1st derivative test
- Find asymptotes, vertical tangents, cusps
- Plot points
- Sketch the curve

(See detailed summary on last page of section 4.3 of Strauss. Page 226 of 5th edition.)

L'Hôpital's Rule

Let f and g be differentiable functions with $g'(x) \neq 0$ on an open interval containing c (except possibly c itself). Suppose

$$\lim_{x \to c} \frac{f(x)}{g(x)}$$

produces an indeterminate form $\frac{0}{0}$ or $\frac{\infty}{\infty}$, and that

$$\lim_{x \to c} \frac{f'(x)}{g'(x)} = L,$$

where L is either a finite number, $+\infty$, or $-\infty$. Then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = L$$

- L'Hôpital's Rule also applies to one-sided limites and to limits to infinity (i.e., where x goes to $+\infty$ or $-\infty$).
- Be careful to check that the limit gives an indeterminate form before applying L'Hôpital's Rule. **Applying it blindly can give you the wrong answer.**

- Note that L'Hôpital's Rule applies only to the indeterminate forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$.
- For other indeterminate forms such as 1^{∞}, 0^0, ∞^0, $\infty - \infty$, and $0\dot{\infty}$, need to manipulate into the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then evaluate with L'Hôpital's Rule.

Special limits with e^x and $\ln x$

If k and n are positive numbers, then

$$\lim_{x \to 0^+} \frac{\ln x}{x^n} = -\infty \qquad \lim_{x \to +\infty} \frac{\ln x}{x^n} = 0$$

$$\lim_{x \to +\infty} \frac{e^{kx}}{x^n} = +\infty \qquad \lim_{x \to +\infty} x^n e^{-kn} = 0$$