

Announcements

- Tuesday 10/30 is the last day to drop
- Homework 9 due Friday 11/2/2007 (posted on web)

Rolle's Theorem: Suppose f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b) . If $f(a) = f(b)$, then there exists at least one number c between a and b such that $f'(c) = 0$.

Mean Value Theorem: If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b) , then there exists in (a, b) at least one number c such that

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

That is, there is at least one point c at which the *average value* equals the *instantaneous change* in f .

Or if f represents the position of a moving object, there is at least one point c at which the *average velocity* equals the *instantaneous velocity*.

Zero-derivative Theorem: If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b) , with $f'(x) = 0$ for all x on (a, b) . Then the function f is constant on $[a, b]$.

Constant Difference Theorem: If f and g are continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b) . Then if $f'(x) = g'(x)$ for all x in (a, b) , there exists a constant C such that

$$f(x) = g(x) + C$$

for all x on $[a, b]$.

Recall:

- The function f is **strictly increasing** on an interval I if

$$f(x_1) < f(x_2) \text{ whenever } x_1 < x_2$$

- The function f is **strictly decreasing** on an interval I if

$$f(x_1) > f(x_2) \text{ whenever } x_1 < x_2$$

for x_1 and x_2 in I .

- f is said to be **strictly monotonic** on an interval I if it is either strictly increasing on all of I or strictly decreasing on all of I .

Monotone function theorem: Let f be differentiable on the open interval (a, b) .

- If $f'(x) > 0$ on (a, b) , then f is strictly increasing on (a, b) .
- If $f'(x) < 0$ on (a, b) , then f is strictly decreasing on (a, b) .

1st Derivative Test for Relative Extrema

1. Find all critical numbers of a continuous function f .
($f'(c) = 0$ or $f'(c)$ does not exist)
2. Classify each critical point as follows:
 - $(c, f(c))$ is a **relative maximum** if
 $f'(x) > 0$ for all x in an open interval (a, c) to the left of c ,
and
 $f'(x) < 0$ for all x in an open interval (c, b) to the right of c
 - $(c, f(c))$ is a **relative minimum** if
 $f'(x) < 0$ for all x in a open interval (a, c) to the left of c ,
and
 $f'(x) > 0$ for all x in an open interval (c, b) to the right of c
 - $(c, f(c))$ is **not an extremum** if the derivative $f'(x)$ has
the same sign for all x in the open intervals (a, c) and (c, b)
on each side of c .

- If the graph of a function f lies above all of its tangents on an interval I , then it is said to be **concave up** on I .
- If the graph of a function f lies below all of its tangents on an interval I , then it is said to be **concave down** on I .
- The graph of a function f is **concave up** on any open interval I where $f''(x) > 0$, and **concave down** where $f''(x) < 0$.
- A point $P(c, f(c))$ on a curve is called an **inflection point** if the graph is concave up on one side of P and concave down on the other side.

2nd Derivative Test for Relative Extrema

Let f be a function such that $f'(c) = 0$ and the second derivative exists on an open interval containing c .

- If $f''(c) > 0$, there is a **relative minimum** at $x = c$.
- If $f''(c) < 0$, there is a **relative maximum** at $x = c$.
- If $f''(c) = 0$, the 2nd derivative test **fails**. (The point could be either a max, a min, or neither.)