Announcements

- Homework 8 due this Friday 10/26/2007
- Thursday 10/30 is the last day to drop

Extreme Values

Let $f(x)$ be a function on an interval I containing number c.

- $f(c)$ is an **absolute maximum** of f on I if $f(c) \geq f(x)$ for all x in I.
- $f(c)$ is an **absolute minimum** of f on I if $f(c) \leq f(x)$ for all x in I.
- The absolute maximum and minimum are called **extreme values** or **absolute extrema** of f on I.
- *Note that a function does not necessarily have a maximum or a minimum on a given interval.*

Extreme Value Theorem: A function f has both an absolute maximum and an absolute minimum on any closed, bounded interval $[a, b]$ where it is continuous.

Note: if f is discontinuous or the interval is not both closed and bounded, we cannot conclude that f has an absolute max and min.

Relative Max and Min

- A function f has a **relative maximum** at a point c if $f(c) \geq f(x)$ for all x in an open interval containing c.
- A function f has a **relative minimum** at a point d if $f(d) \leq f(x)$ for all x in an open interval containing d.
- The relative maxima and minima are called **relative extrema** of f on I.

- Suppose f is defined at c and either $f'(c) = 0$ or $f'(c)$ does not exist. Then c is called a **critical number** of f, and the point $P(c, f(c))$ on the graph is called a **critical point**.
- **Critical number theorem:** If a continuous function f has a relative extremum at c, then c must be a critical number of f. (I.e., either the derivative is 0 or it does not exist at c.)

Procedure for finding absolute extrema

To find the absolute extrema of a continuous function f on $[a, b]$:

1. Compute $f'(x)$ and find all critical numbers of f on $[a, b]$.
   - $f'(x) = 0$
   - $f'(x)$ does not exist
2. Evaluate f at the endpoints a and b and at each critical number c.
3. Compare the values from (2).
   Largest is the absolute maximum of f on $[a, b]$.
   Smallest is the absolute minimum of f on $[a, b]$.

Rolle's Theorem: Suppose f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b). If $f(a) = f(b)$, then there exists at least one number c between a and b such that $f'(c) = 0$.

Mean Value Theorem: If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists in (a, b) at least one number c such that

$$\frac{f(b) - f(c)}{b - a} = f'(c)$$

That is, there is at least one point c at which the *average value* equals the *instantaneous change* in f.

Or if f represents the position of a moving object, there is at least one point c at which the *average velocity* equals the *instantaneous velocity*.

Zero-derivative Theorem: If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), with $f'(x) = 0$ for all x on (a, b). Then the function f is constant on $[a, b]$.

Constant Difference Theorem: If f and g are continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b). Then if $f'(x) = g'(x)$ for all x in (a, b), there exists a constant C such that

$$f(x) = g(x) + C$$

for all x on $[a, b]$.