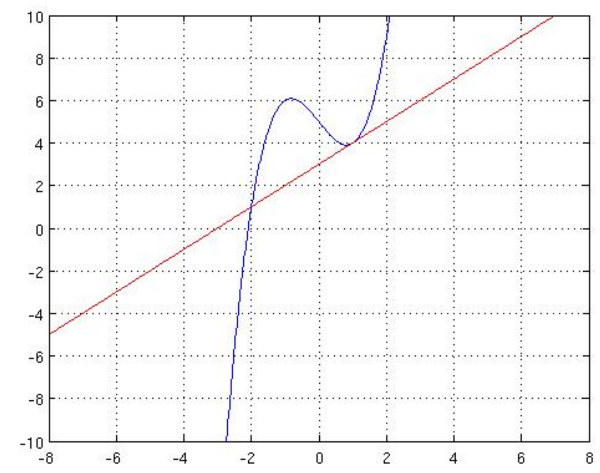
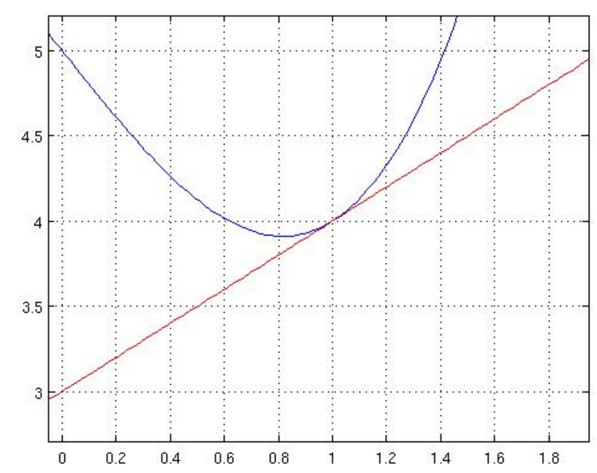
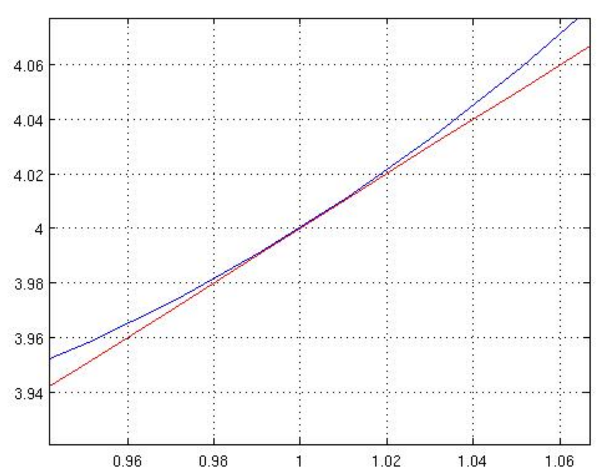
Announcements

- Exam 2 this Friday 10/19/2007
  - Strauss sections 2.4 through 3.7
  - Discussions sections this week are exam review (no quiz)
  - Extra office hours this week:
    Tuesday, 10:00 to 11:30 a.m.
    Wednesday, 10:00 to 11:30 a.m.
    Thursday, 10:00 to 11:30 a.m.
  - Same format as exam 1: in class, no notes, no calculators, etc.
- See extra review problem on course web page
- Homework 8 due Friday 10/26/2007 (will be posted later today)

In the immediate vicinity of P, the tangent line closely approximates the shape of the curve at $y = f(x)$.







Linearization

If x_1 is near a, then $f(x_1)$ is close to the point on the tangent line to $y = f(x)$ at $x = x_1$. That is,

$$f(x_1) \approx f(a) + f'(a)(x_1 - a)$$

This is a *linear approximation* of $f(x)$ at $x = a$. This process is called *linearization* of the function at point $x = a$.

Incremental approximation formula:

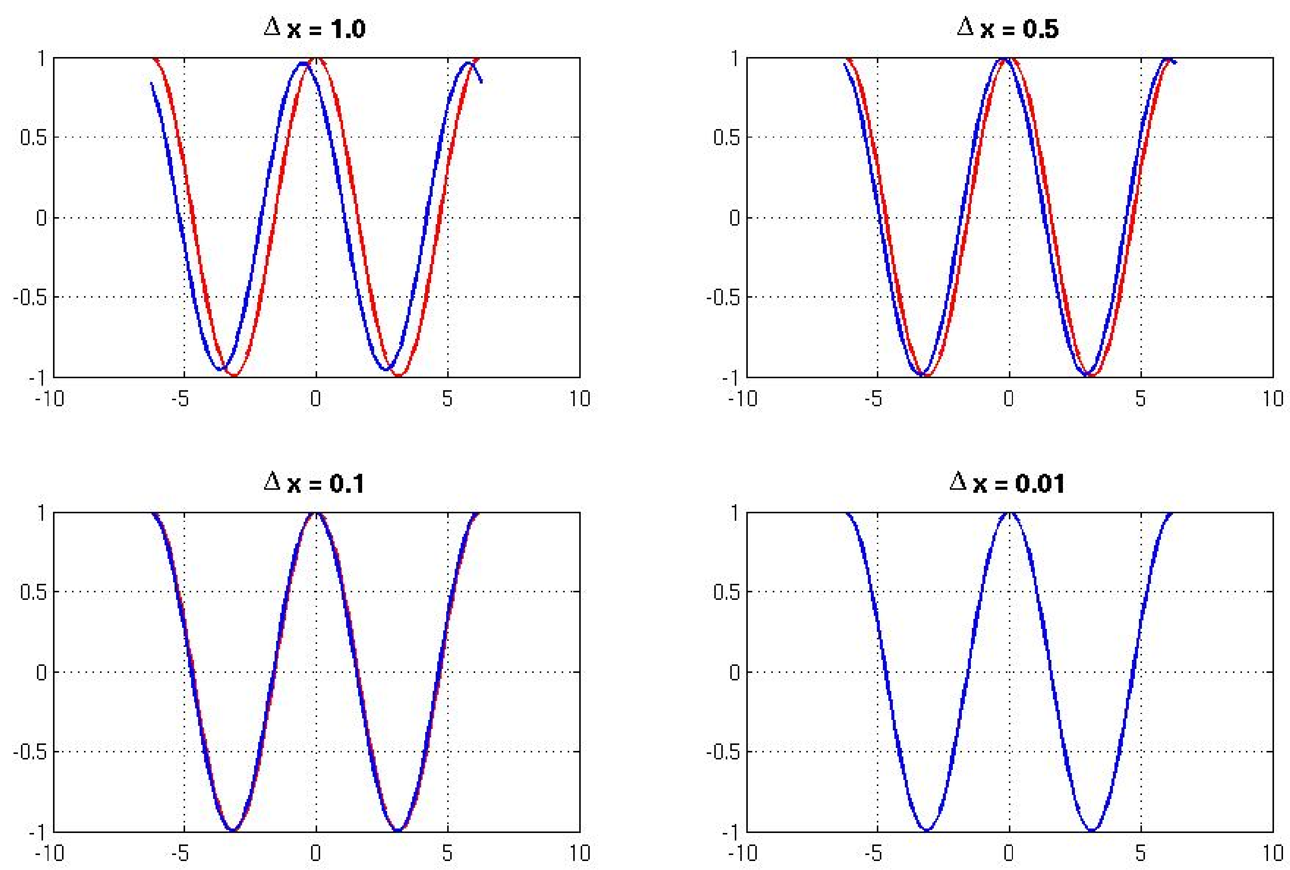
$$f(x_1) - f(a) \approx f'(a)(x_1 - a)$$

$$\Delta y \approx f'(a)\Delta x$$

Incremental approximation of cosine

Let $f(x) = \sin x$. Then $\frac{\Delta f}{\Delta x}$ approximates $f'(x) = \cos x$:

$$\frac{\Delta f}{\Delta x} = \frac{\sin(x_0 + \Delta x) - \sin x_0}{\Delta x} \approx f'(x_0)$$



Differentials

We give the dy and dx of Leibniz notation $\frac{dy}{dx}$ meaning as separate quantities.

dx is called the **differential of x**.

dy is called the **differential of y**.

- $dx = \Delta x$
- But $dy \neq \Delta y$.
  $dy = f'(x)dx$ or equivalently $df = f'(x)dx$.
- Δy is the rise of f that occurs with a change of Δx. But dy is the rise in the *tangent line* relative to the change in x.

Differential rules

Linearity	$d(af + bg) = a\ df + b\ dg$	
Product	$d(fg) = f\ dg + g\ df$	
Quotient	$d(\frac{f}{g}) = \frac{g\ df - f\ dg}{g^2}, (g \neq 0)$	
Power	$d(x^n) = nx^{n-1}\ dx$	
Trig	$d(\sin x) = \cos x\ dx$	$d(\cos x) = -\sin x\ dx$
	$d(\tan x) = \sec^2 x\ dx$	$d(\cot x) = -\csc^2 x\ dx$
	$d(\sec x) = \sec x \tan x\ dx$	$d(\csc x) = -\csc x \cot x\ dx$
Exp and log	$d(e^x) = e^x\ dx$	$d(\ln x) = \frac{1}{x}\ dx$

Differentials for inverse trig

$d(\sin^{-1} u) = \frac{du}{\sqrt{1-u^2}}$ $\quad d(\cos^{-1} u) = \frac{-du}{\sqrt{1-u^2}}$

$d(\tan^{-1} u) = \frac{du}{1+u^2}$ $\quad d(\cot^{-1} u) = \frac{-du}{1+u^2}$

$d(\sec^{-1} u) = \frac{du}{|u|\sqrt{u^2-1}}$ $\quad d(\csc^{-1} u) = \frac{-du}{|u|\sqrt{u^2-1}}$

Exam 2 Review

Trig Identities

- Definitions of trig (and inverse trig) functions
  (SOH CAH TOA)
  Remember that $\sin^{-1} x \neq \frac{1}{\sin x}$.
  $\sin^{-1} x = \arcsin x$
- Pythagorean identities:
  $\cos^2 x + \sin^2 x = 1$
  $1 + \tan^2 x = \sec^2 x$
- Opposite angles (odd vs. even functions)
  $\cos(-x) = \cos(x)$ (i.e., even)
  $\sin(-x) = -\sin(x)$ (i.e, odd)
  $\tan(-x) = -\tan(x)$ (i.e., odd)
- Angle addition:
  $\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$
  $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$