Announcements

- Homework 7 due today.
- Homework 8 will be due Friday 10/26/2007
- Exam 2 this Friday 10/19/2007
  - Strauss sections 2.4 through 3.7
  - Discussions sections this week are exam review (no quiz)
  - Extra office hours this week:
    Tuesday, 10:00 to 11:30 a.m.
    Wednesday, 10:00 to 11:30 a.m.
    Thursday, 10:00 to 11:30 a.m.

Related Rates

Many problems involve a functional relationship $y = f(x)$ in which both x and y are themselves functions of another variable, such as time t.

$$y(t) = f(x(t))$$

We can use implicit differentiation to relate the rate of change $\frac{dy}{dt}$ to the rate $\frac{dx}{dt}$.

General Procedure for Related Rates

1. Draw a figure if appropriate.
2. Assign variables to the quantities that vary.
3. Find a formula or equation that relates the variables.
4. Differentiate the equations (often *implicitly* with respect to time).
5. Substitute specific numerical values where known.
6. Solve algebraically for any required rate.

Examples...

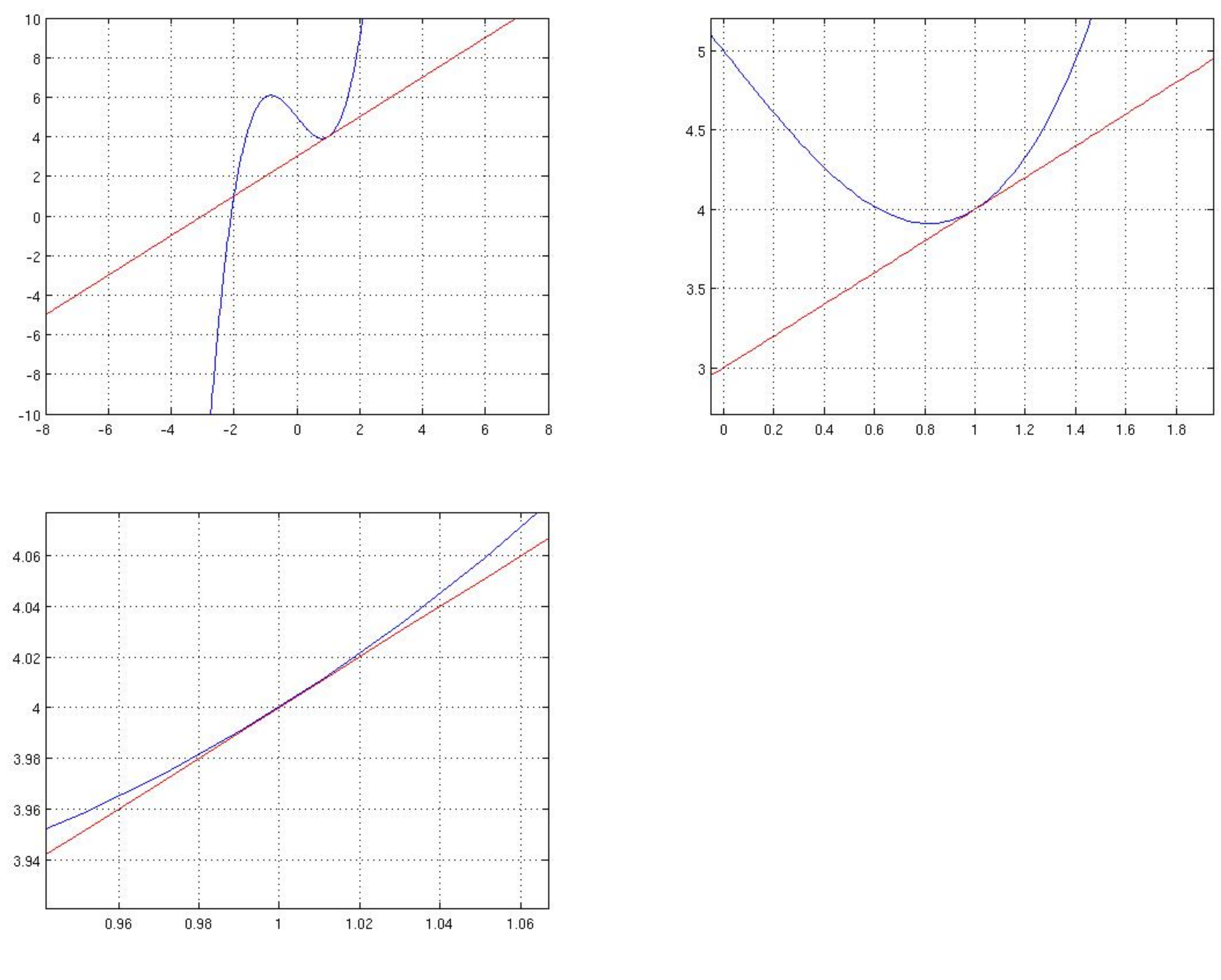
linear approximations and differentials

Recall: if $f(x)$ is differentiable at $x = a$, the tangent line at a point $P(a, f(a))$ on the graph $y = f(x)$ has slope $m = f'(a)$ and equation

$$\frac{y - f(a)}{x - a} = f'(a)$$

or $y = f(a) + f'(a)(x - a)$

In the immediate vicinity of P, the tangent line closely approximates the shape of the curve at $y = f(x)$.



linearization

If x_1 is near a, then $f(x_1)$ is close to the point on the tangent line to $y = f(x)$ at $x = x_1$. That is,

$$f(x_1) \approx f(a) + f'(a)(x_1 - a)$$

This is a *linear approximation* of $f(x)$ at $x = a$. This process is called *linearization* of the function at point $x = a$.

Incremental approximation formula:

$$f(x_1) - f(a) \approx f'(a)(x_1 - a)$$

$$\Delta y \approx f'(a)\Delta x$$

Differentials

We give the dy and dx of Leibniz notation $\frac{dy}{dx}$ meaning as separate quantities.

dx is called the **differential of x**.

dy is called the **differential of y**.

- $dx = \Delta x$
- But $dy \neq \Delta y$.
  $dy = f'(x)dx$ or equivalently $df = f'(x)dx$.
- Δy is the rise of f that occurs with a change of Δx. But dy is the rise in the *tangent line* relative to the change in x.

Differential rules

Linearity	$d(af + bg) = a\ df + b\ dg$	
Product	$d(fg) = f\ dg + g\ df$	
Quotient	$d(\frac{f}{g}) = \frac{g\ df - f\ dg}{g^2}, (g \neq 0)$	
Power	$d(x^n) = nx^{n-1}\ dx$	
Trig	$d(\sin x) = \cos x\ dx$	$d(\cos x) = -\sin x\ dx$
	$d(\tan x) = \sec^2 x\ dx$	$d(\cot x) = -\csc^2 x\ dx$
	$d(\sec x) = \sec x \tan x\ dx$	$d(\csc x) = -\csc x \cot x\ dx$
Exp and log	$d(e^x) = e^x\ dx$	$d(\ln x) = \frac{1}{x}\ dx$

Differentials for inverse trig

$d(\sin^{-1} u) = \frac{du}{\sqrt{1-u^2}}$ $d(\cos^{-1} u) = \frac{-du}{\sqrt{1-u^2}}$

$d(\tan^{-1} u) = \frac{du}{1+u^2}$ $d(\cot^{-1} u) = \frac{-du}{1+u^2}$

$d(\sec^{-1} u) = \frac{du}{|u|\sqrt{u^2-1}}$ $d(\csc^{-1} u) = \frac{-du}{|u|\sqrt{u^2-1}}$