Announcements

- Homework 6 due today.
- Homework 7 due next Monday, 10/15/2007
- Exam 2 on Friday 10/19/2007
  - Strauss sections 2.4 through 3.8

Implicit Differentiation

If an equation defines y implicitly as a differentiable function of x, find $\frac{dy}{dx}$ by:

1. Differentiate both sides of equation with respect to x. (Remember that y is really a function of x for part of the curve and use the chain rule when differentiating terms containing y.)
2. Solve the differentiated equation algebraically for $\frac{dy}{dx}$.

warning

Beware that implicit differentiation is only valid if y is a differentiable function of x. Can get errors if you apply it carelessly.

Example: $x^2 + y^2 = -1$

Differentiation formulas for inverse trig functions

$\frac{d}{dx}(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$ $\frac{d}{dx}(\cos^{-1} u) = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$

$\frac{d}{dx}(\tan^{-1} u) = \frac{1}{1+u^2}\frac{du}{dx}$ $\frac{d}{dx}(\cot^{-1} u) = \frac{-1}{1+u^2}\frac{du}{dx}$

$\frac{d}{dx}(\sec^{-1} u) = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$ $\frac{d}{dx}(\csc^{-1} u) = \frac{-1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$

Derivatives of exponential and logarithmic functions with base b

Let u be a differentiable function of x, and b be a positive number (other than 1). Then

$$\frac{d}{dx}b^u = (\ln b)b^u\frac{du}{dx}$$

$$\frac{d}{dx}\log_b u = \frac{1}{\ln b}\cdot\frac{1}{u}\frac{du}{dx}$$

Derivative of $\ln|u|$

If $f(x) = \ln|x|$, $x \neq 0$, then $f'(x) = \frac{1}{x}$.

Also, if u is a differentiable function of x, then

$$\frac{d}{dx}\ln|u| = \frac{1}{u}\frac{du}{dx}$$

Logarithmic Differentiation

Using logarithms, we can trade differentiating products and quotients for differentiating sums and differences.

Useful to handling complicated product or quotient functions and exponential functions where variables appear in both the base and the exponent.

Take logarithm of both side, then apply logarithm rules to simplify, then differentiate.

Related Rates

Many problems involve a functional relationship $y = f(x)$ in which both x and y are themselves functions of another variable, such as time t.

$$y(t) = f(x(t))$$

We can use implicit differentiation to relate the rate of change $\frac{dy}{dt}$ to the rate $\frac{dx}{dt}$.

General Procedure for Related Rates

1. Draw a figure if appropriate.
2. Assign variables to the quantities that vary.
3. Find a formula or equation that relates the variables.
4. Differentiate the equations (often *implicitly* with respect to time).
5. Substitute specific numerical values where known.
6. Solve algebraically for any required rate.