Announcements

- Homework 6 due next Monday, 10/8/2007

Theorem: Chain Rule

If $y = f(x)$ is a differentiable function of u, and u is a differentiable function of x, then $y = f(u(x))$ is a differentiable function of x and its derivative is given by

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Theorem: Alternative form of Chain Rule

If u is differentiable at x and f is differentiable at $u(x)$, then the composite function $f \circ u$ is differentiable at x and

$$\frac{d}{dx} f[u(x)] = \frac{d}{du} f(u) \frac{du}{dx}$$

Extended derivative formulas

If u is a differentiable function of x, then

Extended Power Rule

$$\frac{d}{dx}u^n = nu^{n-1}\frac{du}{dx}$$

Extended Trig Rules

$\frac{d}{dx}\sin u = \cos u\frac{du}{dx}$ $\frac{d}{dx}\cos u = -\sin u\frac{du}{dx}$

$\frac{d}{dx}\tan u = \sec^2 u\frac{du}{dx}$ $\frac{d}{dx}\cot u = -\csc^2 \frac{du}{dx}$

$\frac{d}{dx}\sec u = \sec u\tan u\frac{du}{dx}$ $\frac{d}{dx}\csc u = -\csc u\cot u\frac{du}{dx}$

Extended Exponential and Logarithmic Rules

$$\frac{d}{dx}e^u = e^u\frac{du}{dx} \qquad \frac{d}{dx}\ln u = \frac{1}{u}\frac{du}{dx}$$

Implicit Differentiation

If an equation defines y implicitly as a differentiable function of x, find $\frac{dy}{dx}$ by:

1. Differentiate both sides of equation with respect to x. (Remember that y is really a function of x for part of the curve and use the chain rule when differentiating terms containing y.)
2. Solve the differentiated equation algebraically for $\frac{dy}{dx}$.

warning

Beware that implicit differentiation is only valid if y is a differentiable function of x. Can get errors if you apply it carelessly.

Example: $x^2 + y^2 = -1$

Differentiation formulas for inverse trig functions

$\frac{d}{dx}(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$ $\frac{d}{dx}(\cos^{-1} u) = \frac{-1}{\sqrt{1-u^2}} \frac{du}{dx}$

$\frac{d}{dx}(\tan^{-1} u) = \frac{1}{1+u^2} \frac{du}{dx}$ $\frac{d}{dx}(\cot^{-1} u) = \frac{-1}{1+u^2} \frac{du}{dx}$

$\frac{d}{dx}(\sec^{-1} u) = \frac{1}{|u|\sqrt{u^2-1}} \frac{du}{dx}$ $\frac{d}{dx}(\csc^{-1} u) = \frac{-1}{|u|\sqrt{u^2-1}} \frac{du}{dx}$

Derivatives of exponential and logarithmic functions with base b

Let u be a differentiable function of x, and b be a positive number (other than 1). Then

$$\frac{d}{dx} b^u = (\ln b) b^u \frac{du}{dx}$$

$$\frac{d}{dx} \log_b u = \frac{1}{\ln b} \cdot \frac{1}{u} \frac{du}{dx}$$

Derivative of $\ln|u|$

If $f(x) = \ln|x|$, $x \neq 0$, then $f'(x) = \frac{1}{x}$.

If u is a differentiable function of x, then

$$\frac{d}{dx}\ln|u| = \frac{1}{u}\frac{du}{dx}$$