## Announcements

- Homework 6 due next Monday, 10/8/2007
  Will be posted on course web page today.

## Using derivatives in graphing

We already know how to find $x$ and $y$ intercepts.

Now we can also find places where the graph turns: $\frac{dy}{dx} = 0$.

Example: graph $y = (x^2 + 4x - 7)$

Example: graph $y = (x - 2)(x^2 + 4x - 7)$

## Derivatives of trigonometric functions

**Theorem:** The functions $\sin x$ and $\cos x$ are differentiable for all $x$ and

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = \sin x$$

## More Trig review

Recall:

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

$$\sec x = \frac{1}{\cos x} \qquad \csc x = \frac{1}{\sin x}$$

$$\cos^2 x + \sin^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\cot^2 x + 1 = \csc^2 x$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

## Derivatives of trigonometric functions

The trigonometric functions $\sin, \cos, \tan, \csc, \sec, \cot$ are all differentiable wherever they are defined, and

$$\begin{aligned} \tfrac{d}{dx}\sin x &= \cos x \qquad & \tfrac{d}{dx}\cos x &= -\sin x \\ \tfrac{d}{dx}\tan x &= \sec^2 x \qquad & \tfrac{d}{dx}\cot x &= -\csc^2 x \\ \tfrac{d}{dx}\sec x &= \sec x \tan x \qquad & \tfrac{d}{dx}\csc x &= -\csc x \cot x \end{aligned}$$

## Derivatives of exponentials and logarithms

The natural exponential $e^x$ is differentiable for all $x$, with derivative

$$\frac{d}{dx}e^x = e^x$$

(The slope of the graph of the natural expontial at any point $c$ is $e^c$.)

The natural logarithm $\ln x$ is differentiable for all $x > 0$, with derivative

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

## Rates of Change

$$\text{Average rate of change } = \frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\text{Instantaneous rate of change } = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Suppose $f(x)$ is differentiable at $x = x_0$. Then the **instantaneous rate of change** of $y = f(x)$ with respect to $x$ at $x_0$ is

$$f(x_0) = \left.\frac{dy}{dx}\right|_{x=x_0}$$

The **relative rate of change** of $y = f(x)$ at $x = x_0$ is given by the ratio

$$\frac{\text{Instantaneous rate of change}}{\text{size of the quantity}} = \frac{f'(x_0)}{f(x_0)}$$