Announcements

- Homework 5 due Friday 9/28/07
- Exam 1 will be returned Friday

Derivatives

The **derivative** of f at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

provided the limit exists.

Techniques of Integration

Theorem: The derivative of a constant is zero.

$$\frac{d}{dx}(k) = 0$$

Theorem: For any real number n, the power function $f(x) = x^n$ has derivative $f'(x) = nx^{n-1}$, i.e.,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Basic rules for combining derivatives

Rule	**Function Notation**
Constant multiple	$[cf(x)]' = cf'(x)$
Sum	$[f(x) + g(x)]' = f'(x) + g'(x)$
Difference	$[f(x) - g(x)]' = f'(x) - g'(x)$
Linearity	$[af(x) + bg(x)]' = af'(x) + bg'(x)$
Product	$[f(x)g(x)]' = f(x)g'(x) + f'(x)g(x)$
Quotient	$[\frac{f(x)}{g(x)}]' = \frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$

Rule	**Leibniz Notation**
Constant multiple	$\frac{d}{dx}(cf) = c\frac{df}{dx}$
Sum	$\frac{d}{dx}(f+g) = \frac{df}{dx} + \frac{dg}{dx}$
Difference	$\frac{d}{dx}(f-g) = \frac{df}{dx} - \frac{dg}{dx}$
Linearity	$\frac{d}{dx}(af+bg) = a\frac{df}{dx} + b\frac{dg}{dx}$
Product	$\frac{d}{dx}(fg) = f\frac{dg}{dx} + g\frac{df}{dx}$
Quotient	$\frac{d}{dx}(\frac{f}{g}) = \frac{g\frac{df}{dx} - f\frac{dg}{dx}}{g^2}$

Extended linearity rule

If $f_1, f_2, \ldots, f_n$ are differentiable functions and $a_1, a_2, \ldots, a_n$ are constants, then

$$\frac{d}{dx}[a_1 f_1 + a_2 f_2 + \cdots + a_n f_n] = a_1 \frac{df_1}{dx} + a_2 \frac{df_2}{dx} + \cdots + a_n \frac{df_n}{dx}$$

Higher-order derivatives

1st derivative	y'	$f'(x)$		$\frac{dy}{dx}$	$\frac{d}{dx}f(x)$
2nd derivative	y''	$f''(x)$	$\frac{d}{dx}(\frac{dy}{dx}) =$	$\frac{d^2y}{dx^2}$	$\frac{d^2}{dx^2}f(x)$
3rd derivative	y'''	$f'''(x)$	$\frac{d}{dx}(\frac{d^2y}{dx^2}) =$	$\frac{d^3y}{dx^3}$	$\frac{d^3}{dx^3}f(x)$
4th derivative	$y^{(4)}$	$f^{(4)}(x)$		$\frac{d^4y}{dx^4}$	$\frac{d^4}{dx^4}f(x)$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
nth derivative	$y^{(n)}$	$f^{(n)}(x)$		$\frac{d^ny}{dx^n}$	$\frac{d^n}{dx^n}f(x)$

Derivatives of trigonometric functions

Theorem: The functions $\sin x$ and $\cos x$ are differentiable for all x and

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = \sin x$$

More Trig review

Recall:

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$
$$\sec x = \frac{1}{\cos x} \qquad \csc x = \frac{1}{\sin x}$$
$$\cos^2 x + \sin^2 x = 1$$
$$1 + \tan^2 x = \sec^2 x$$
$$\cot^2 x + 1 = \csc^2 x$$
$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

Derivatives of trigonometric functions

The functions $\sin, \cos, \tan, \csc, \sec, \cot$ are all differentiable wherever they are defined, and

$$\begin{aligned} \tfrac{d}{dx}\sin x &= \cos x & \tfrac{d}{dx}\cos x &= -\sin x \\ \tfrac{d}{dx}\tan x &= \sec^2 x & \tfrac{d}{dx}\cot x &= -\csc^2 x \\ \tfrac{d}{dx}\sec x &= \sec x \tan x & \tfrac{d}{dx}\csc x &= -\csc x \cot x \end{aligned}$$