Announcements

- Homework 5 due Friday 9/28/07
- Quizzes resume this week
- Exam 1 will be returned Wednesday or Friday

Compounding of interest

Suppose a sum of money is invested in an account where interest is compounded n times per year with annual interest rate r. Then after one period (one compounding),

$$A_1 = P(1 + \frac{r}{n})$$

After two periods:

$$A_2 = A_1(1 + \frac{r}{n}) = P(1 + \frac{r}{n})^2$$

After t years, interest has been compounded nt times and the future value is

$$A(t) = P(1 + \frac{r}{n})^{nt}$$

Continuously compounded interest

Often interest is compounded *instantaneously*. In this case,

$$\begin{aligned} A(t) &= \lim_{n\to\infty} P(1+\frac{r}{n})^{nt} \\ &= Pe^{rt} \end{aligned}$$

Differentiation

For a function $f(x)$, the slope of a **secant line** passing through points $P(x_0, f(x_0))$ and $Q((x_0 + \Delta x), f(x_0 + \Delta x))$ is

$$m = \frac{\Delta y}{\Delta x} = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

At the point $P(x_0, f(x_0))$, the **tangent line** to the graph of f has slope given by

$$\lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

provided the limit exists.

Derivative

The **derivative** of f at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

provided the limit exists.

- The function f is **differentiable at** x if the limit of the difference quotient exists
- A function is not necessarily differentiable throughout its entire domain.
- Notice that the derivative of a function is itself a function.

Theorem: If f is differentiable at x_0, the graph of $y = f(x)$ has a **tangent line** at the point $P(x_0, y_0)$ with slope $f'(x_0)$ and equation

$$y = f'(x_0)(x - x_0) + f(x_0).$$

Prove using point-slope formula for a line.

The **normal line** to the graph of f at the point P is the line that is perpendicular to the tangent line to the graph at point P.

Theorem: If a function f is differentiable at a point c, then it is also continuous at c.

- That is, *differentiability implies continuity.*
- The converse is not true. If a function f is continuous at c, it may or may not be differentiable at c.
- If a function is discontinuous at c, then it does not have a derivative at c.

Notation

For the function $y = f(x)$, the derivative can be written several ways:

$$f'(x) = \frac{dy}{dx} = y'$$

The notation $\frac{dy}{dx}$ is called *Leibniz notation*. When evaluated at a point $x = c$ we can write:

$$f'(c) = \left.\frac{dy}{dx}\right|_{x=c}$$

E.g., if $y = f(x) = x^2$, the derivative at $x = c$ can be written as

$$f'(c) = \left.\frac{df}{dx}\right|_{x=c} = \left.\frac{dy}{dx}\right|_{x=c} = \left.\frac{d}{dx}(x^2)\right|_{x=c} = 2c$$