Announcements

- HW 4 due **today**. Solutions will be posted this afternoon.
- Homework 5 due Friday 9/28/07 (posted on web today)
- Three typos on practice questions:
  (5 a) should be $y - 2x + 7 = 0$
  (8 a) should be $f(x) = x^2 + 3x + 1$
  (14) should have $f(x) = 2x^3 + x + 1$
- **Exam 1 on this Friday, 9/21**
  - Exam 1 will cover: Chapter 1 (all 4 sections) and Chapter 2, sections 2.1, 2.2, and 2.3
  - It will be a written exam (no scantron, no multiple choice)
  - Show all work.
  - There is partial credit.
  - No calculators, no electronics, no notes, no neighbors.
  - Exam review in Sections this week (no quiz).
  - Extra office hours Thursday 10:00 to 11:30 a.m.

Exam tips

- Do problems easiest for you first.
- Don't get stuck spending too much time on one problem.
- Show your work. I am testing that you know how to approach the problem, not just that you get the right answer.
- Mark your final answer clearly (circle, box, etc.), especially if you try several things first.

Exponentials and Logarithms

Let x be a real number, and let r_n be a sequence of rational numbers such that

$$x = \lim_{x \to \infty} r_n$$

Then the **exponential function** with base $b > 0$ $(b \neq 1)$ is given by

$$b^x = \lim_{n \to \infty} b^{r_n}$$

$y = b^x$ $(b > 0,\ b \neq 1)$ is monotonic, so the exponential has an inverse.

If $b > 0$ and $b \neq 1$, the **logarithm of x to the base b** is the function $y = \log_b x$ that satisfies $b^y = x$.

$$y = log_b x \ \text{ means } \ b^y = x$$

Natural exponential and natural logarithm

$$e = \lim_{n\to\infty}(1+\frac{1}{n})^n$$

$$e \approx 2.7182818\cdots$$

- Notation: $exp(x) = e^x$

- **Common Logarithm** is the log base 10, $\log_{10} x$, and is written $\log x$
- **Natural Logarithm** is the log base e, $\log_e x$, and is written $\ln x$
- **Beware:** the notation "$\log x$" is often used to represent whatever is the standard base in a particular field (or a particular course).
- But "$\ln x$" always means the Natural Logarithm.
- In this class, we'll write the common logarithm as $\log x$ or $\log_{10} x$, the natural logarithm as $\ln x$ or $\log_e x$, and any other base as $\log_b x$.

Basic properties of the natural logarithm:
(logarithm base e)

- $\ln 1 = 0$
- $\ln e = \log_e e = 1$
- $e^{\ln x} = x$ for all $x > 0$
- $\ln e^y = y$ for all y
- $b^x = e^{x \ln b}$ for any $b > 0$ $(b \neq 1)$

Change of basis theorem:

$$\log_b x = \frac{\ln x}{\ln b}$$

for any $b > 0$ ($b \neq 1$)

Important, for example, for taking logarithms on a calculator or in many software packages, which often only do common and natural logarithms.

What is so natural about the natural log?

Many important growth (and decay) processes are described in terms of natural exponentials and logarithms. For example,

- Can be used to describe some types of biological colony growth e.g., exponential growth of E. Coli bacteria
- Describes continuous compound interest
- Newton's law of cooling: $T = e^{kt+c} + R$, where T is the temperature of an object after time t of being placed in an environment of temperature R. k is a constant.
- Many other applications such as spread of disease, radioactive decay...

Example: Exponential growth

A colony grows such that at time t its population is $P(t)$:

$$P(t) = P_0 e^{kt}$$

where P_0 is the initial population and k is a positive constant.

If the colony begins with 5000 individuals and has a population of 7000 after 20 minutes, what is the constant k, and what will the population be after 30 minutes?

Continuous compounding of interest

Suppose a sum of money is invested. If interest is compounded once in a period, the future value A after one period is

$$A = P + Pi = P(1 + i)$$

where P is the principal (amount invested) and Pi is the interest earned after one period.

Now suppose interest is compounded n times per year with annual interest rate r. Then after one period (one compounding),

$$A_1 = P(1 + \frac{r}{n})$$

After two periods:

$$A_2 = A_1(1 + \frac{r}{n}) = P(1 + \frac{r}{n})^2$$

After t years, interest has been compounded nt times and the future value is

$$A(t) = P(1 + \frac{r}{n})^{nt}$$

Often interest is compounded *instantaneously*. In this case,

$$\begin{aligned} A(t) &= \lim_{n \to \infty} P(1 + \frac{r}{n})^{nt} \\ &= Pe^{rt} \end{aligned}$$