Announcements

- Homework 4 due **Wednesday 9/19**
  Section 2.3: 6, 10, 12, 16, 24, 28, 30, 38, 40
  Bonus: Section 2.2: 59, 63; Section 2.3: 46
  (bonus problems worth up to 5 pts each; up to 15 pts total)
- **Exam 1 on this Friday, 9/21**
- Exam 1 will cover:
  - Chapter 1 (all 4 sections)
  - Chapter 2, sections 2.1, 2.2, and 2.3
- Exam review in Sections this week (no quiz)
- Extra office hours this week:
  Tuesday 10:00 to 11:30 a.m.
  Thursday 10:00 to 11:30 a.m.

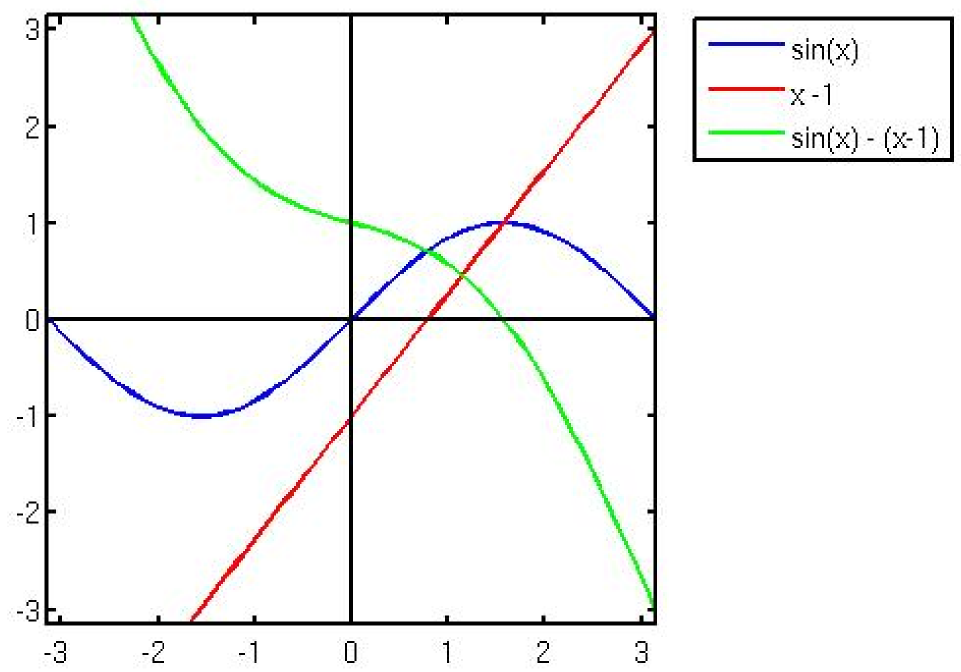
Intermediate Value Theorem

If f is a continuous function on the closed interval $[a, b]$ and L is some number strictly between $f(a)$ and $f(b)$, then there exists at least one number c on the open interval (a, b) such that $f(c) = L$.

(In other words: if f is a continuous function on $[a, b]$ then $f(x)$ must take on all values between $f(a)$ and $f(b)$.)

Root location theorem

If f is continuous on the closed interval $[a, b]$ and if $f(a)$ and $f(b)$ have opposite algebraic signs, then $f(c) = 0$ for at least one number c on the open interval (a, b).



Show that $f(x) = \sin x - x + 1$ has a root on the interval $(\frac{\pi}{2}, \pi)$:

Proof:

- $f(x)$ is continuous on the interval $[\frac{\pi}{2}, \pi]$.
- $f(\frac{\pi}{2}) = 1 - \frac{\pi}{2} + 1 = 2 - \frac{\pi}{2} > 0$
- $f(\pi) = 0 - \pi + 1 < 0$
- Therefore $f(x)$ has at least one root in $(\frac{\pi}{2}, \pi)$.

Exponentials and Logarithms

Recall: for n a natural number (positive integer) and b a real number,

$$b^n = b \cdot b \cdot b \cdots b \;\; \text{(n times)}$$

If $b \neq 0$, then $b^0 = 1$, $b^{-n} = \frac{1}{b^n}$.

And if $b > 0$, then $b^{1/n} = \sqrt[n]{b}$ and $b^{m/n} = \sqrt[n]{b^m}$ for integers m and n and $\frac{m}{n}$ a reduced fraction.

Completeness property of the reals

For any real number x, there exist rational numbers r_n such that

$$x = \lim_{n\to\infty} r_n$$

That is, for any number $\epsilon > 0$, there exists a number N such that $|x - r_n| < \epsilon$ whenever $n > N$.

Exponential function

Let x be a real number, and let r_n be a sequence of rational numbers such that

$$x = \lim_{x \to \infty} r_n$$

Then the **exponential function** with base $b > 0$ $(b \neq 1)$ is given by

$$b^x = \lim_{n \to \infty} b^{r_n}$$

Properties of the exponential function

Let x and y be real numbers, and a and b be positive real numbers.

Equality rule	If $b \neq 1$, then $b^x = b^y$ if and only if $x = y$
Inequality rules	If $x > y$ and $b > 1$, then $b^x > b^y$
	If $x > y$ and $0 < b < 1$, then $b^x < b^y$
Product rule	$b^x b^y = b^{x+y}$
Quotient rule	$\frac{b^x}{b^y} = b^{x-y}$
Power rules	$(b^x)^y = b^{xy}$; $(ab)^x = a^x b^x$; $(\frac{a}{b})^x = \frac{a^x}{b^x}$

Logarithmic function

$y = b^x$ ($b > 0$, $b \neq 1$) is monotonic, so the exponential has an inverse.

If $b > 0$ and $b \neq 1$, the **logarithm of x to the base b** is the function $y = \log_b x$ that satisfies $b^y = x$.

$$y = log_b x \quad \text{means} \quad b^y = x$$

Properties of the logarithmic function

Let x and y be real numbers, and assume $b > 0$ and $b \neq 1$.

Equality rule	$\log_b x = \log_b y$ if and only if $x = y$
Inequality rules	If $x > y$ and $b > 1$, then $\log_b x > \log_b y$
	If $x > y$ and $0 < b < 1$, then $\log_b x < \log_b y$
Product rule	$\log_b(xy) = \log_b x + \log_b y$
Quotient rule	$\log_b(\frac{x}{y}) = \log_b x - \log_b y$
Power rules	$\log_b x^p = p \log_b x$ for any real number p
Inversion rules	$b^{\log_b x} = x$ and $\log_b b^x = x$
Special values	$\log_b b = 1$ and $\log_b 1 = 0$

Natural exponential and natural logarithm

$$e = \lim_{n\to\infty}(1+\frac{1}{n})^n$$

$$e \approx 2.7182818\cdots$$

- Notation: $exp(x) = e^x$

- **Common Logarithm** is the log base 10, $\log_{10} x$, and is written $\log x$
- **Natural Logarithm** is the log base e, $\log_e x$, and is written $\ln x$
- **Beware:** the notation "$\log x$" is often used to represent whatever is the standard base in a particular field (or a particular course).
- But "$\ln x$" always means the Natural Logarithm.
- In this class, we'll write the common logarithm as $\log x$ or $\log_{10} x$, the natural logarithm as $\ln x$ or $\log_e x$, and any other base as $\log_b x$.

Basic properties of the natural logarithm:

- $\ln 1 = 0$
- $\ln e = 1$
- $e^{\ln x} = x$ for all $x > 0$
- $\ln e^y = y$ for all y
- $b^x = e^{x \ln b}$ for any $b > 0$ $(b \neq 1)$