Announcements

- Homework 3 due today.
- Exam 1 on Friday 9/21 - one week from today
- Exam 1 will cover:
  - Chapter 1 (all 4 sections)
  - Chapter 2, sections 2.1, 2.2, and 2.3
- Exam review in Sections next week (no quiz)
- Homework 4 posted today; due **Wednesday 9/19**
  Section 2.3: 6, 10, 12, 16, 24, 28, 30, 38, 40
  Bonus: Section 2.2: 59, 63; Section 2.3: 46
  (bonus problems worth up to 5 pts each; up to 15 pts total)

Recall: formal definition of continuity at a point

A function f is **continuous at a point x = c** if the following three conditions are satisfied:

1. $f(c)$ is defined
2. $\lim_{x\to c} f(x)$ exists
3. $\lim_{x\to c} f(x) = f(c)$

The key idea of continuity is that if x is close to c, then $f(x)$ is close to $f(c)$.

Continuity Theorems

Theorem: If f is a polynomial, rational function, power function, trigonometric function, or an inverse trigonometric function, then f is continuous at any number $x = c$ for which $f(c)$ is defined.

Theorem: If functions f and g are continuous at $x = c$, then the following functions are also continuous at $x = c$:

Scalar multiple	sf	for any constant (scalar) s
Sum and difference	$f + g$ and $f - g$	
Product	fg	
Quotient	$\frac{f}{g}$	provided $g(c) \neq 0$
Composition	$f \circ g$	provided g cont at c and f cont at $g(c)$

Composition Limit Rule:

If $\lim_{x\to c} g(x) = L$ and f is continuous at L, then $\lim_{x\to c} f[g(x)] = f(L)$. I.e.,

$$\lim_{x\to c} f[g(x)] = f(L) = f(\lim_{x\to c} g(x))$$

Applies similarly to left and right limits.

Idea is that the limit of a continuous function is the function of the limiting value.

Continuity from the left and right:

The function f is **continuous from the right at** a if and only if

$$\lim_{x \to a^+} f(x) = f(a)$$

The function f is **continuous from the left at** b if and only if

$$\lim_{x \to b^-} f(x) = f(b)$$

Continuity on an interval:

The function f is **continuous on the open interval** (a, b) if it is continuous at each number in the interval. (Note that the end points are not in the interval.)

If f is continuous on (a, b) and also continuous from the right at a, then f is **continuous on the half-open interval** $[a, b)$.

If f is continuous on (a, b) and also continuous from the left at b, then f is **continuous on the half-open interval** $(a, b]$.

If f is continuous on (a, b), continuous from the right at a, and continuous from the left at b, then f is **continuous on the closed interval** $[a, b]$.

Checking continuity at "suspicious points"

Often there are only a few points in the domain of a function where a discontinuity my occur. We call such points **suspicious points**:

- where the defining rule for f changes (e.g., piecewise defined functions, absolute values, etc.)
- where substitution of $x = c$ causes division by 0

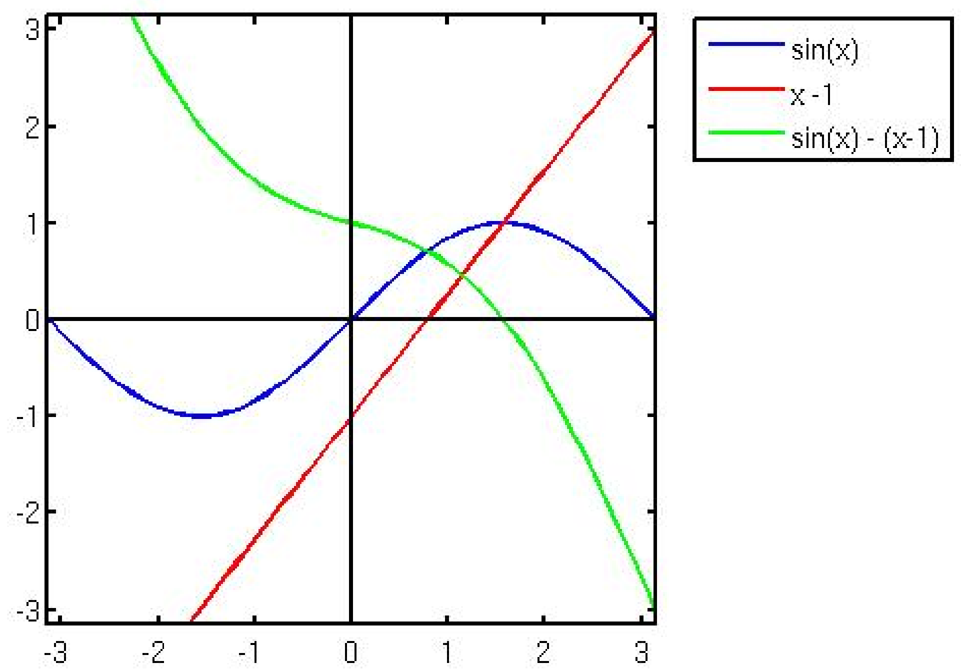
Intermediate Value Theorem

If f is a continuous function on the closed interval $[a, b]$ and L is some number strictly between $f(a)$ and $f(b)$, then there exists at least one number c on the open interval (a, b) such that $f(c) = L$.

(In other words: if f is a continuous function on $[a, b]$ then $f(x)$ must take on all values between $f(a)$ and $f(b)$.)

Root location theorem

If f is continuous on the closed interval $[a, b]$ and if $f(a)$ and $f(b)$ have opposite algebraic signs, then $f(c) = 0$ for at least one number c on the open interval (a, b).



Show that $f(x) = \sin x - x + 1$ has a root on the interval $(\frac{\pi}{2}, \pi)$:

Proof:

- $f(x)$ is continuous on the interval $[\frac{\pi}{2}, \pi]$.
- $f(\frac{\pi}{2}) = 1 - \frac{\pi}{2} + 1 = 2 - \frac{\pi}{2} > 0$
- $f(\pi) = 0 - \pi + 1 < 0$
- Therefore $f(x)$ has at least one root in $(\frac{\pi}{2}, \pi)$.