Announcements

- Homework 3 due Friday 9/14
- Exam 1 on Friday 9/21

Some examples of techniques for limits

- Fractional reduction
- Rationalization
- Piecewise functions

Squeeze Rule

(a.k.a. sandwich rule)

If $g(x) \leq f(x) \leq h(x)$ for all $x \neq c$ in an open interval about c, and if

$$\lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L,$$

then

$$\lim_{x \to c} f(x) = L$$

Some special limits

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \to 0} \frac{\cos x - 1}{x} = 0$$

- Example: use the Squeeze Rule to prove:

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

Continuity

Intuitively, continuity means without jumps or breaks.

Formal definition:

A function f is **continuous at a point** $\mathbf{x = c}$ if the following three conditions are satisfied:

1. $f(c)$ is defined
2. $\lim_{x \to c} f(x)$ exists
3. $\lim_{x \to c} f(x) = f(c)$

A function that is not continuous at c is said to have a **discontinuity** at that point.

The key idea of continuity is that if x is close to c, then $f(x)$ is close to $f(c)$.

Continuity Theorems

Theorem: If f is a polynomial, rational function, power function, trigonometric function, or an inverse trigonometric function, then f is continuous at any number $x = c$ for which $f(c)$ is defined.

Theorem: If functions f and g are continuous at $x = c$, then the following functions are also continuous at $x = c$:

Scalar multiple	sf	for any constant (scalar) s
Sum and difference	$f + g$ and $f - g$	
Product	fg	
Quotient	$\frac{f}{g}$	provided $g(c) \neq 0$
Composition	$f \circ g$	provided g cont at c and f cont at $g(c)$