Announcements

- Homework 3 due Friday
- Exam 1 on Friday 9/21
- One volunteer note taker needed

Informal limit definition:
The notation

$$\lim_{x \to c} f(x) = L$$

means that the function values $f(x)$ can be made arbitrarily close to a unique number L by choosing x sufficiently close to c (but not equal to c).

Notation: Also written as $f(x) \to L$ as $x \to c$

Limits do not always exist:

If the limit of the function f fails to exist, $f(x)$ is said to **diverge** as $x \to c$.

- The function may grow arbitrarily large (or small) as $x \to c$
  E.g., $\lim_{x\to 0} \frac{1}{x^2}$

A function f that increases or decreases without bound as x approaches c is said to **tend to infinity** (inf) as $x \to c$.

$$\lim_{x\to c} f(x) = +\infty \qquad \text{if f increases without bound}$$
$$\lim_{x\to c} f(x) = -\infty \qquad \text{if f decreases without bound}$$

- The function may oscillate as $x \to c$
  E.g., $\lim_{x\to 0} \sin\frac{1}{x}$
  divergence by oscillation

Formal definition of a limit

The limit statement

$$\lim_{x \to c} f(x) = L$$

means that for each number $\epsilon > 0$, there corresponds a number $\delta > 0$ with the property that

$$|f(x) - L| < \epsilon \quad \text{whenever} \quad 0 < |x - c| < \delta$$

(also called "epsilon-delta" definition of a limit)

Basic properties and rules for limits

For any real number c, suppose f and g both have limits at $x = c$:

$$\begin{aligned}
\text{Contant rule} &: \lim_{x\to c} k = k \\
\text{Limit of x rule} &: \lim_{x\to c} x = c \\
\text{Multiple rule} &: \lim_{x\to c}[kf(x)] = k\lim_{x\to c} f(x) \\
\text{Sum rule} &: \lim_{x\to c}[f(x)+g(x)] = \lim_{x\to c} f(x) + \lim_{x\to c} g(x) \\
\text{Difference rule} &: \lim_{x\to c}[f(x)-g(x)] = \lim_{x\to c} f(x) - \lim_{x\to c} g(x) \\
\text{Product rule} &: \lim_{x\to c}[f(x)g(x)] = [\lim_{x\to c} f(x)][\lim_{x\to c} g(x)] \\
\text{Quotient rule} &: \lim_{x\to c}\frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)} \\
&\ \ \text{if } \lim_{x\to c} g(x) \neq 0 \\
\text{Power rule} &: \lim_{x\to c}[f(x)]^n = [\lim_{x\to c} f(x)]^n \\
&\ \ \text{(for n rational and limit on right exists)}
\end{aligned}$$

Limit of a polynomial: If P is a polynomial function, then

$$\lim_{x\to c} P(x) = P(c)$$

Limit of a rational function: If Q is a rational function $Q(x) = \frac{P(x)}{D(x)}$ then

$$\lim_{x\to c} Q(x) = \frac{P(c)}{D(c)}$$

provided $\lim_{x\to c} D(x) \neq 0$

Limits of trigonometric functions:

If c is any number in the domain of the given function, then

$$\lim_{x \to c} \cos x = \cos c \qquad \lim_{x \to c} \sec x = \sec c$$
$$\lim_{x \to c} \sin x = \sin c \qquad \lim_{x \to c} \csc x = \csc c$$
$$\lim_{x \to c} \tan x = \tan c \qquad \lim_{x \to c} \cot x = \cot c$$

- Fractional reduction
- Rationalization
- Piecewise functions