

Announcements

- Please write your Section number on your homework.
750 = Tuesday 12:30 p.m.
751 = Thursday 12:30 p.m.
752 = Tuesday 5:00 p.m.
753 = Thursday 5:00 p.m.
- Homework 3 will be posted today (due next Friday)
- One volunteer note taker needed.
- Exam 1 on Friday 9/21 (two weeks)

Graphing f^{-1}

- If (a, b) is a point on the graph of f , then (b, a) is a point on the graph of f^{-1} .
- You can graph f^{-1} by reflecting the graph of f about the line $x = y$.
- Can think of it as graphing f , flipping the paper, then rotating 90° clockwise.

Example of proving inverse trig identities

Example: For $-1 \leq x \leq 1$, show that $\cos(\sin^{-1} x) = \sqrt{1 - x^2}$

Solution: Let $\alpha = \sin^{-1}(x)$, where $-\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$

Solve with reference triangle.

Chapter 2: Limit of a function

Consider a free falling body with no air resistance. Falls approximately $s(t) = 16t^2$ feet in t seconds.

Previously, we looked at *average velocity* over an interval of time.

Now we want to know **instantaneous velocity** at $t = 2$ seconds.

We can express this as a limit.

We'll compute the average velocity over a smaller and smaller time interval near $t = 2$ seconds. Let \bar{v} be the average velocity over the interval $1.9 \leq t \leq 2$.

$$\begin{aligned}\bar{v} &= \frac{\text{distance traveled}}{\text{elapsed time}} \\ &= \frac{s(2) - s(1.9)}{2 - 1.9} \\ &= \frac{16(2)^2 - 16(1.9)^2}{0.1} \\ &= 62.4 \text{ ft/s}\end{aligned}$$

We can make similar computations with smaller intervals from above or below $t = 2$.

Int.	[1.9,2]	[1.99,2]	[1.999,2]		[2,2.0001]	[2,2.001]	[2,2.01]
Len.	0.1	0.01	0.001		0.0001	0.001	0.01
v	62.4	63.84	63.98		64.0016	64.016	64.16

Average velocity seems to be approaching 64 as we make the time intervals smaller. So we expect that the instantaneous velocity when $t = 2$ will be 64.

Average velocity of the falling body over time interval $2 \leq t \leq 2 + h$ is

$$\frac{s(2+h) - s(2)}{(2+h) - 2} = \frac{16(2+h)^2 - 16(2)^2}{h}$$

In this example, the average velocity has a limiting value of 64 as the length h of the time interval tends to zero:

$$\lim_{h \rightarrow 0} \frac{16(2+h)^2 - 16(2)^2}{h} = 64$$

Informal limit definition:

The notation

$$\lim_{x \rightarrow c} f(x) = L$$

means that the function values $f(x)$ can be made arbitrarily close to a unique number L by choosing x sufficiently close to c (but not equal to c).

Notation: Also written as $f(x) \rightarrow L$ as $x \rightarrow c$

Example: Evaluate (by table)

$$L = \lim_{x \rightarrow 0} \frac{2\sqrt{x+1} - x - 2}{x^2}$$

x	-0.5	-0.1	-0.01	-0.001	0	0.001	0.005
f(x)	-0.3431	-0.2633	-0.2513	-0.2501	undef	-0.2499	-0.2494

Table pattern suggests that the limit L is -0.25 .

Be careful with calculators (or computers) and very small numbers.
Can introduce errors.

One-Sided Limits

Right-hand limit:

$$\lim_{x \rightarrow c^+} f(x) = L$$

if we can make $f(x)$ arbitrarily close to L by choosing x sufficiently close to c on a small interval (c, b) immediately to the right of c .

Left-hand limit:

$$\lim_{x \rightarrow c^-} f(x) = L$$

if we can make $f(x)$ arbitrarily close to L by choosing x sufficiently close to c on a small interval (a, c) immediately to the left of c .

Theorem: (one-sided limit theorem)

The two-sided limit $\lim_{x \rightarrow c} f(x)$ exists if and only if the two one-sided limits exist and are equal. Furthermore, if

$$\lim_{x \rightarrow c^-} f(x) = L = \lim_{x \rightarrow c^+} f(x),$$

then

$$\lim_{x \rightarrow c} f(x) = L$$

Limits do not always exist:

If the limit of the function f fails to exist, $f(x)$ is said to **diverge** as $x \rightarrow c$.

- The function may grow arbitrarily large (or small) as $x \rightarrow c$
E.g., $\lim_{x \rightarrow 0} \frac{1}{x^2}$

A function f that increases or decreases without bound as x approaches c is said to **tend to infinity** (inf) as $x \rightarrow c$.

$$\lim_{x \rightarrow c} f(x) = +\infty \quad \text{if } f \text{ increases without bound}$$

$$\lim_{x \rightarrow c} f(x) = -\infty \quad \text{if } f \text{ decreases without bound}$$

- The function may oscillate as $x \rightarrow c$
E.g., $\lim_{x \rightarrow 0} \sin \frac{1}{x}$

divergence by oscillation