

Announcements

- Homework 1 due today.
- Homework 2 due next Friday in class.
- In class quizzes during discussion sections begin next week.

Section 1.3 — functions and graphs

- **Definitions:** A **function** f is a rule that assigns to each element x of a set X a unique element y of a set Y . The element y is called the **image** of x under f and is denoted $f(x)$. The set X is called the **domain** of f , and the set of all images of elements of X is called the **range** of the function.
- A function f can be thought of as the set of ordered pairs (x, y) where each member x of the domain is associated with exactly one member $y = f(x)$ of the range.
- A function assigns a unique “output” to each legitimate “input”

- If the range of f consists of all of Y , then f is said to map X **onto** Y .
- If each element in the range is the image of one and only one element in the domain, then f is said to be a **one-to-one** function.
- A function f is **bounded** on $[a, b]$ if there exists a number B such that $|f(x)| \leq B$ for all x in $[a, b]$.
- In a function represented as $y = f(x)$, x and y are called **variables**. Since y is determined by x , y is called the **dependent variable** and x is called the **independent variable**.
- Example: $f(x) = x^2 + 4x + 5$ is the set of all ordered pairs (x, y) satisfying $y = x^2 + 4x + 5$.

- **Evaluating** a function means to find the value of f for a particular value in the domain.

- Example: $f(x) = 2x^2 - x$

Find $f(x + h)$

Find $\frac{f(x+h)-f(x)}{h}$,

where x and h are real numbers and $h \neq 0$.

- $\frac{f(x+h)-f(x)}{h}$ is called a **difference quotient**. It will be important in chapter 3 (derivatives).

Piece-wise defined functions

- Some functions are defined differently on different parts of their domain.
- Example:

$$f(x) = \begin{cases} x \sin x & \text{if } x < 2 \\ 3x^2 + 1 & \text{if } x \geq 2 \end{cases}$$

- Find $f(\frac{\pi}{2})$ and $f(2)$:

Application

An object dropped from a height in a vacuum will fall a distance of s feet in t seconds according to the formula:

$$s(t) = 16t^2, t \geq 0$$

1. How far with the object fall in the first second?
In the next 2 seconds?
2. How far with it fall during the time interval $t = 1$ sec to $t = 1 + h$ sec?
3. What is the average rate of change of distance during the time $t = 1$ sec to $t = 3$ sec?
4. What is the average rate of change of distance during the time $t = x$ sec to $t = x + h$ sec?

- **Domain** of f is assumed to be the set of real numbers for which the function is defined (unless otherwise specified).

- Examples: find the domain:

$$f(x) = 2x - 1$$

$$g(x) = 2x - 1, \quad x \neq -3$$

$$h(x) = \frac{(2x-1)(x+3)}{x+3}$$

$$F(x) = \sqrt{x+2}$$

$$G(x) = \frac{4}{5-\cos x}$$

- **Function equality:** Two functions f and g are **equal** if and only if
 1. f and g have the same domain
 2. $f(x) = g(x)$ for all x in the domain