

## Announcements

- **Course web page:**  
[www.math.ttu.edu/~vhowle/Courses/Math1351-Fall2007.html](http://www.math.ttu.edu/~vhowle/Courses/Math1351-Fall2007.html)
- **Phone problems:** Voicemail not working on my phone. If I don't answer, please try email.
- **Tutoring:** The Missouri Club offers free tutoring beginning 9/4. Hours are Mon. through Fri. 10:00 to 5:00 in MA 106.
- **TA office hours:** Yining (Summer) Du's office hours are Tues. & Thurs. 2:00 – 4:00 in MA 250. Her phone is 742-2580.
- **Textbook:** Strauss 5th edition is in at Double T (and possibly other stores by now). But 4th edition is fine if you have that.
- **Notes:** Today is last day of providing comprehensive notes.

## Distance between points in the plane

- **Theorem:** distance between two points  $P_1(x_1, y_1)$  and  $P_2(x_2, y_2)$  is given by:

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- Proof uses Pythagorean theorem and absolute value properties.

## Midpoint formula

$$M \left( \frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2} \right)$$

- Average 1st and 2nd components of coordinates of the endpoints.
- (Typo in Monday's notes)

## Graph of an equation

- Graph of eqn in two variables  $x$  and  $y$  is the collection of all points  $P(x, y)$  whose coordinates  $(x, y)$  satisfy the equation.

- Example: Find eqn of a circle with center  $(h, k)$  and radius  $r$ .

Solution:

Let  $(x, y)$  be any point on the circle.

$r =$  distance from  $(h, k)$  to  $(x, y)$

$$r = \sqrt{(x - h)^2 + (y - k)^2}$$

$$r^2 = (x - h)^2 + (y - k)^2$$

- Unit circle:  $x^2 + y^2 = 1$
- Example: Find circle with center  $(3, -5)$  that passes through the point  $(1, 8)$

Solution:  $r = \sqrt{(1 - 3)^2 + (8 - (-5))^2} = \sqrt{(4 + 169)} = \sqrt{173}$

$$(x - 3)^2 + (y + 5)^2 = 173$$

## Graphing a circle from its equation

- Example:  $4x^2 + 4y^2 - 4x + 8y - 5 = 0$

Solution:

$$4x^2 + 4y^2 - 4x + 8y - 5 = 0$$

$$x^2 + y^2 - x + 2y = 5/4$$

$$(x^2 - x) + (y^2 + 2y) = \frac{5}{4}$$

$$\left(x^2 - x + \left(-\frac{1}{2}\right)^2\right) + (y^2 + 2y + 1^2) = \frac{5}{4} + \frac{1}{4} + 1$$

$$\left(x - \frac{1}{2}\right)^2 + (y + 1)^2 = \frac{5}{2}$$

Circle with radius  $\sqrt{5/2}$ , center at  $(\frac{1}{2}, -1)$

## Trigonometry

- degrees and radians

$$\frac{\theta(deg)}{360} = \frac{\theta(rad)}{2\pi}$$

- Some exact trig values:

Func / Angle $\theta$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	1/2	0
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined

- In this course,  $\theta$  is in radians unless it is written as  $\theta^\circ$ ,  
e.g.,  $\sin \frac{\pi}{2} = \sin 90^\circ = 1$

- Example: Solve  $\sin x + \sqrt{3} \cos x = 1$  on  $[0, 2\pi)$ .

Solution:

$$\sqrt{3} \cos x = 1 - \sin x$$

$$3 \cos^2 x = 1 - 2 \sin x + \sin^2 x$$

$$3(1 - \sin^2 x) = 1 - 2 \sin x + \sin^2 x$$

$$2 + 2 \sin x - 4 \sin^2 x = 0$$

$$2 \sin^2 x - \sin x - 1 = 0$$

$$(2 \sin x + 1)(\sin x - 1) = 0$$

$$\sin x = -\frac{1}{2} \quad \sin x = 1$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6} \quad x = \frac{\pi}{2}$$

Check for extraneous roots since we squared both sides.

$$\sin x + \sqrt{3} \cos x = 1 \quad \text{on} \quad [0, 2\pi)$$

- $7\pi/6$ :

$$\begin{aligned} \sin\left(\frac{7\pi}{6}\right) + \sqrt{3} \cos\left(\frac{7\pi}{6}\right) &= -\frac{1}{2} + \sqrt{3}\left(-\frac{\sqrt{3}}{2}\right) \\ &= -\frac{1}{2} - \frac{3}{2} \neq 1 \end{aligned}$$

- $11\pi/6$ :

$$\begin{aligned} \sin\left(\frac{11\pi}{6}\right) + \sqrt{3} \cos\left(\frac{11\pi}{6}\right) &= -1/2 - \sqrt{3}\left(\frac{\sqrt{3}}{2}\right) \\ &= -\frac{1}{2} + \frac{3}{2} = 1 \end{aligned}$$

- $\pi/2$ :

$$\sin\left(\frac{\pi}{2}\right) + \sqrt{3} \cos\left(\frac{\pi}{2}\right) = 1 - \sqrt{3}(0) = 1$$

$$x = \frac{\pi}{2}, \frac{11\pi}{6}$$

## Section 1.2

- Lines have a constant slope,  $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$  for  $\Delta x \neq 0$
- A nonvertical line that contains the points  $P(x_1, y_1)$  and  $Q(x_2, y_2)$  has slope

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

- The **angle of inclination** of a line L is the angle  $\phi$ , ( $0 \leq \phi < \pi$ ), between L and the positive x-axis.  
The **slope** of L with inclination  $\phi$  is  $m = \tan \phi$ 
  - Derived directly from slope of a line through two points  $P(x_1, y_1)$  and  $Q(x_2, y_2)$ , where  $\phi$  is angle of inclination, and definition of  $\tan \phi$  (opposite over adjacent).
- Line L is **rising** if  $0 < \phi < \frac{\pi}{2}$  and **falling** if  $\frac{\pi}{2} < \phi < \pi$ . Slope is undefined ( $\tan \phi$  is undefined) for  $\phi = \frac{\pi}{2}$ .

## Forms of linear equations

- **Standard form:**  $Ax + By + C = 0$   
( $A, B, C$  constants;  $A$  and  $B$  not both zero)
- **Slope-intercept form:**  $y = mx + b$   
(slope  $m$ , y-intercept  $(0, b)$ ).
- **Point-slope form:**  $y - k = m(x - h)$   
(slope  $m$ , through point  $(h, k)$ )
- **Two-intercept form:**  $\frac{x}{a} + \frac{y}{b} = 1$   
(intercepts  $(a, 0)$  and  $(0, b)$ ).
- **Horizontal line:**  $y = k$  (slope 0)
- **Vertical line:**  $x = h$  (slope undefined)

Be able to graph a line from any of the forms, convert between the forms, and find the equation of a line from two points, from one point and a slope, from its graph.

### Slope criteria for parallel and perpendicular lines

- If  $L_1$  and  $L_2$  are nonvertical lines with slopes  $m_1$  and  $m_2$ , then  
 $L_1$  and  $L_2$  are **parallel** *if and only if*  $m_1 = m_2$   
 $L_1$  and  $L_2$  are **perpendicular** *if and only if*  
 $m_1 m_2 = -1$ , i.e.,  $m_1 = -\frac{1}{m_2}$
- This says that nonvertical and nonhorizontal lines are **parallel** if and only if their slopes are equal. They are **perpendicular** if and only if their slopes are negative reciprocals of each other.

- **Example: find eqns of parallel and perpendicular lines**

Let  $L$  be the line  $3x + 2y = 5$

- Find an eqn of the line parallel to  $L$  passing through point  $P(4, 7)$  and write in standard form:

Rewrite eqn for  $L$  in slope-intercept form:  $y = -\frac{3}{2}x + \frac{5}{2}$

Therefore slope of  $L$  (and parallel lines) is  $m = -\frac{3}{2}$

Use point-slope form to find eqn:  $y - 7 = -\frac{3}{2}(x - 4)$

Rewrite in standard form:

$$2y - 14 = -3x + 12$$

$$3x + 2y - 26 = 0$$

- Find an eqn of the line perpendicular to  $L$  passing through point  $P(4, 7)$  and write in standard form:

A line perpendicular to  $L$  must have slope  $\frac{2}{3}$  (neg reciprocal)

Use point-slope form:  $y - 7 = \frac{2}{3}(x - 4)$

Rewrite in standard form:

$$3y - 12 = 2x - 8$$

$$2x - 3y + 4 = 0$$