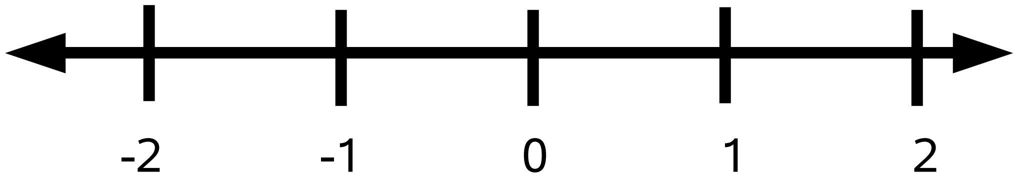
Math 1351-011

- Syllabus
- Distance on a number line, ordering
- Absolute value
- Distance in the plane & midpoint formula
- Homework 1

Distance on a number line

- **Real numbers**
- **Real number line**



  - Distance on a number line
    - Distance from x to the origin
    - Distance between two points on the line
- **Order properties**: real numbers a,b,c,d
  - **Trichotomy law**: exactly one of the following is true
    $a < b$, $a > b$, or $a = b$
  - **Transitive law of inequality**: if $a < b$ and $b < c$, then $a < c$
  - **Additive law of inequality**: if $a < c$ and $b < d$, then $a+b < c+d$
  - **Multiplicative law of inequality**:
    if $a < b$, then $ac < bc$ if $c > 0$,
    and $ac > bc$ if $c < 0$

Absolute value

- **|a|** $= \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$
- The number x is located |x| units from 0
  - To the right if $x > 0$; to the left if $x < 0$
- **Distance between numbers** x_1 and x_2 is $|x_1 - x_2|$
  - Note that $|x_1 - x_2| = |x_2 - x_1|$
- **Terminology**: "p *if and only if* q" means that both the statement and its converse are true, i.e.,

  "if p, then q" and "if q, then p"

Properties of the absolute value

For a,b real numbers:

- $|a| \geq 0$ absolute value is nonnegative
- $|-a| = |a|$
- $|a|^2 = a^2$
- $|ab| = |a|\,|b|$
- $|a/b| = |a| / |b|$, $b \neq 0$
- $-|a| \leq a \leq |a|$ true because either $|a| = a$ or $|a| = -a$
- Let $b \geq 0$; $|a| = b$ *if and only if* $a = \pm b$
- Let $b > 0$; $|a| < b$ *if and only if* $-b < a < b$
- Let $b > 0$; $|a| > b$ if and only if $a > b$ or $a < -b$ (think of number line)
- $|a + b| \leq |a| + |b|$ **Triangle Inequality (very useful in theory and computation)**

Interval notation

Closed interval	$a \le x \le b$	**[a,b]**	
	$a \le x$	**[a,∞)**	
	$x \le b$	**(-∞, b]**	
Open interval	$a < x < b$	**(a,b)**	
	$a < x$	**(a, ,∞)**	
	$x < b$	**(-∞, b)**	
Half-open	$a < x \le b$	**(a,b]**	
	$a \le x < b$	**[a,b)**	
Real number line		**(-∞,∞)**	

Absolute value equations

Example: solve $|2x - 6| = x$

Solution:

if $2x - 6 \geq 0$, then $|2x - 6| = 2x - 6$

solve $2x - 6 = x$

$x = 6$

if $2x - 6 < 0$, then $|2x - 6| = -(2x - 6)$

solve $-(2x - 6) = x$

$-3x = -6$

$x = 2$

Two solutions: $x = 6$ and $x = 2$

(It is always a good idea to plug them back in and doubled check!)

Recall that $|a - b|$ is the distance between a and b on the number line. So $|x - a| = b$ is satisfied by the two points x that are of distance b from a. The equation above is satisfied by the two points x such that the distance between 2x and 6 on the number line is x.

Absolute value inequality

Example: Solve $|2x - 3| \leq 4$

Solution:

$-4 \leq 2x - 3 \leq 4$ (using one of our abs value properties)

$-4 + 3 \leq 2x \leq 4 + 3$

$-1 \leq 2x \leq 7$

$-1/2 \leq x \leq 7/2$

$[-1/2, 7/2]$ in interval notation

See textbook for geometric solution

Absolute value as tolerance

- Let w be a measurement (e.g., weight)
- $|w - a| \le b$ can be interpreted as “w being compared to a with **absolute error** of measurement of b units.”
- Example: a bag of cement weighs 90 lbs plus or minus 2 lbs
  So a given bag can weigh as much as 92 lbs or as little as 88 lbs.
  State as an absolute value inequality.
- Solution: let w = weight of the bag of cement in pounds
  $90 - 2 \le w \le 90 + 2$
  $-2 \le w - 90 \le 2$
  $|w - 90| \le 2$
-

Distance between points in the plane

- Theorem: distance between two points $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ is given by $d=\sqrt{((\Delta x)^2+(\Delta y)^2)}=\sqrt{((x_2-x_1)^2+(y_2-y_1)^2)}$

  Δx = horizontal change $x_2 - x_1$ (a.k.a. "run")

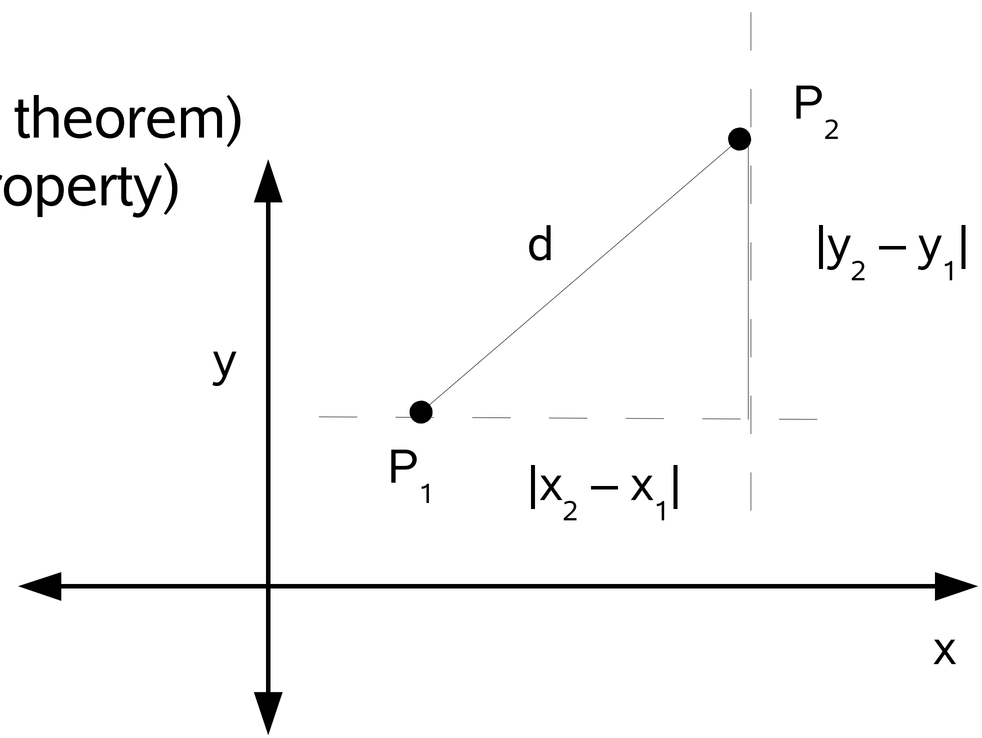
  Δy = vertical change $y_2 - y_1$ (a.k.a. "rise")

- Proof

  $d^2 = |\Delta x|^2 + |\Delta y|^2$ (Pythagorean theorem)

  $d^2 = (\Delta x)^2 + (\Delta y)^2$ (abs value property)

$$d=\sqrt{((\Delta x)^2+(\Delta y)^2)}$$



Midpoint formula

- Midpoint of a line segment with endpoints $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ has coordinates

$$M\left(\frac{(x1+x2)}{2},\frac{(x1+x2)}{2}\right)$$

Average 1st and 2nd components of coordinates of endpoints.