The world's first baby announcement /math problem set!

Today's problem set is to compute the masses and heights of two very cute babies:

Russell Stewart Howle
Charlotte Morgan Howle

who were born the morning of 30 December 2008 at Baptist St. Anthony's Hospital in Amarillo Texas. They were so anxious to explore the world they unexpectedly arrived just over ten weeks early! Russell arrived about 1 minute before Charlotte. Both are doing well, but because of the early arrival they will be in NICU for at least several weeks.

Parents Vicki Howle and Kevin Long are delighted, and are dealing as well as could be expected with the early arrival. Mostly they're trying to figure out where even to *begin* when asked "so, any questions?" by the nursing staff! They are also trying to figure out the logistics of looking after babies in NICU in Amarillo while teaching 120 miles away in Lubbock; with luck, the babies will be able to be transferred to the Lubbock NICU very soon!

The babies were carried by Vicki's wonderful cousin, Cindy Looby. Cindy is recovering quickly after many complications and a tumultuous early delivery. Words cannot express our gratitude to Cindy, her husband Lanny, and their son Cameron for all they've gone through to bring these little ones into the world. It's been a long and unexpectedly complicated process, and we've been supported throughout by our families and friends.

Pictures!

So here they are! Charlotte is on the left, Russell on the right. These are through glass walls of a NICU unit so there are some reflections. Russell is on a breathing tube and is being kept in a high humidity environment; the foggy walls made it difficult to get a picture except from one awkward angle.

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In[1]:= c1 = {Import["~/Kids/Charlie2.jpg", ImageSize → Small],
   Import["~/Kids/Russ2.jpg", ImageSize → Small]}
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Out[1]= {, }

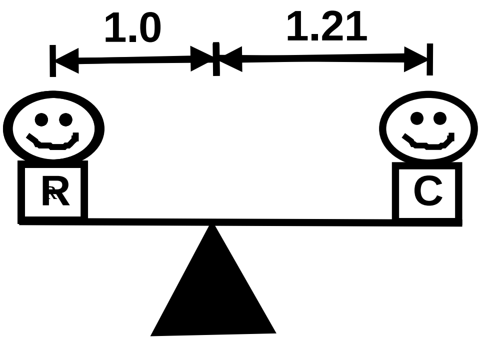
And now what you're *really* waiting for: math problems!

Every baby announcement should include several practice problems. If you've forgotten systems of equations and the physics of simple machines, or if you're just too lazy to work through a couple of problems, **the answers are at the end, highlighted in red.**

■ Problem 1

Archimedes performs two experiments. First, he puts the two babies in a boat and notices that the boat now displaces 2.7 liters of water beyond the boat's unladen displacement. Second, he puts Charlotte and Russell on a see−saw and finds that in order to balance the two babies, Charlotte must sit 21% farther from the fulcrum than Russell. (See the diagram below)

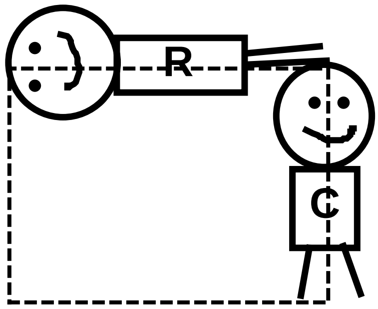
Given this information, calculate the mass of each baby.



■ Problem 2

Placing the two babies at right angles to one another defines a rectangle with perimeter 1.55 meters and area 0.15015 square meters.
The diagram below shows the configuration of the babies.

From this information, calculate the height of each baby.



Solutions

Problem 1

Long ago Archimedes discovered that a floating object will displace its own weight in water (his famous "Eureka" moment). When the babies sit in a floating boat, they displace 2.7 additional liters. The density of water is one kilogram per liter, so the mass of the water displaced is 2.7 kilograms. According to Archimedes' Principle, the total mass of the two babies is therefore 2.7 kilograms.

Archimedes did experiments on many other mechanical phenomena; for example, he also found that two objects on a scale (or seesaw) will balance when the ratio of the lever arms is inversely proportional to the ratio of the masses. Because Charlie must sit 21% further from the fulcrum than Russ to balance, it must be that $M_R = 1.21\, M_C$.

We've therefore found two equations relating the masses:

$M_R + M_C = 2.7\,\text{kg}$
$M_R - 1.21\, M_C = 0$

These are two linear equations in two unknowns which can be solved by eliminating one of the variables ("Gaussian elimination"). For example, subtract the second equation from the first; Russ' mass cancels out, leaving
$2.21\, M_C = 2.7\,\text{kg}$. Therefore, Charlie's mass is $M_C = 2.7\,\text{kg} / 2.21 = 1.22\,\text{kg}$.

Having solved for M_C, substitute $M_C = 1.22$ kg into the second equation to find $M_R = 1.47$ kg.

So Charlotte has mass 1.22 kg, and Russell has mass 1.47 kg. In English units, that's **2 lb, 11 oz for Charlotte and 3 lb, 4 oz for Russell**.

(In addition to being the greatest physicist of the ancient world, Archimedes was also the father of computational mathematics. He invented the first superlinear algorithm, the "method of exhaustion" for computing approximations to π .)

■ Problem 2

The problem of determining the sides of a rectangle from knowledge of its perimeter and area is a very old one: it appears in an ancient Egyptian papyrus in the context of finding the dimensions of a plot of land. This was the first recorded use of a quadratic equation.

The two equations are $A = h_1 h_2$ and $P = 2(h_1 + h_2)$. Substituting $h_1 = A/h_2$ into the second equation gives $P = 2(A/h_2 + h_2)$. Multiply both sides by $h_2/2$, finding $h_2{}^2 - \frac{1}{2} P h_2 + A = 0$.

This is a quadratic equation for h_2. Notice that we'd have found exactly the same equation -- but for h_1 -- had we eliminated h_2 instead of h_1. The same equation gives us *both* heights. It should therefore have two solutions -- one for each baby's height -- and sure enough, quadratic equations always have two solutions. Here goes (given $P = 1.55\,m$ and $A = 0.15015\,m^2$ as specified).

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Solve[{h^2 - P h / 2 + A == 0}, h] /. {P → 1.55, A → 0.15015}
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$\{\{h \to 0.385\}, \{h \to 0.39\}\}$

Aren't computers wonderful! The babies' heights are 0.385 and 0.39 meters, or **15 $\frac{1}{8}$ and 15 $\frac{3}{8}$ inches**. Russell is the taller of the two, by $\frac{1}{4}$ inch.