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Model of a multiple-lens, single-fiber system in a compound eye

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Abstract. In earlier work, we showed that a simple apposition compound eye can form the hardware basis for a new type of inertial navigation system, that can be used on micro air vehicles. One of the key properties of a simple apposition eye is that it is a low pass, spatial filter. This property had been shown in mathematical models based on Fourier optics for a single lens and single fiber system. However, a simple apposition compound eye usually consists of thousands of lenses called ommatidia. An important question that has not been studied is the effect the presence of multiple lenses have on the angular sensitivity, or in some other words, the bandwidth of the low pass, spatial filter. In this paper, we develop a Fourier optics model for a multiple lens, single-fiber system. We obtain expressions for the intensity distribution near the focus of a single lens due to the presence of neighboring lenses and for the angular sensitivity of the system. Numerical simulations show a less than 10 percent increase in the angular sensitivity, even when five or more lenses are considered to focus light on a single rhabdom in a worker bee and an artificial eye.

Keywords: Simple apposition compound eye, Fourier optics, multiple lenses, angular sensitivity

1. Introduction

Compound eyes are characterized by the presence of several eyelets ("ommatidia") each of which consists of a corneal surface, a lens and an optical fiber called "rhabdom". Recently, we showed in [2,3] that it is possible to build a strap-down type inertial navigation system (that we call an Optical inertial Navigation System (ONS)) using a simple apposition eye, that is a type of compound eye found in insects. Specifically, we showed that sequences of images from a simple apposition compound eye can be processed to yield angular and linear velocities (in body coordinates) for a micro air vehicle. The key property of a simple apposition eye that makes this application possible is that the lens-optical fiber combination found in a single ommatidium acts as a low pass, spatial filter. Due to this, an image formed by such a device is necessarily blurred, or in some other words, locally smoothed.

The spatial, low pass filtering property of a single ommatidium is directly linked to its small angular sensitivity, which is about 1.4 degrees from the axis of the lens for a worker honey bee (*Apis Mellifera*) and about 2.2 degrees for the artificial eye of Jeong, Kim and Lee [4]. Using Stavenga's model [8] for a single ommatidium, and after carefully selecting parameters, we were able to show that the theoretically predicted angular sensitivity compares very well with experimental observations. The experimental data on a single ommatidium were obtained by Jeong, Kim and Lee [4] for a worker honey bee and for

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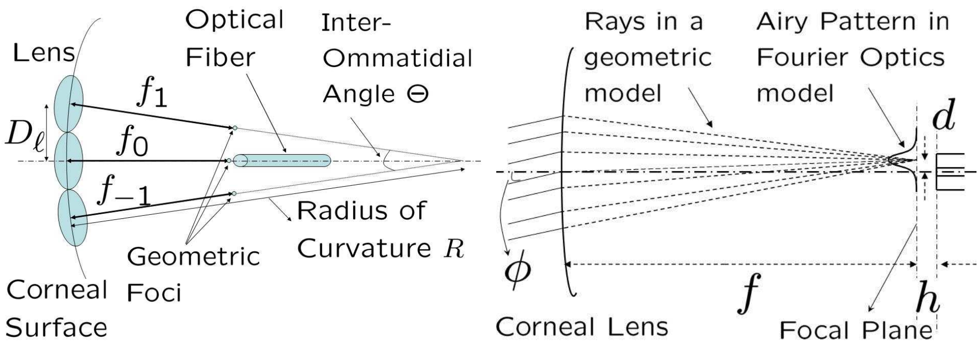
an artificial eye. However, the actual compound eye consists of several lenses in close proximity; for example, six lenses can be found sharing a boundary with a central lens in *Apis Mellifera*. In this paper, we investigate the effect of the neighboring lenses have on the angular sensitivity of the ommatidium. To simplify computations, we assume spherical symmetry which allows us to work with the scalar wave equation and use the techniques of Fourier optics. We derive expressions for the intensity of light in the region between a lens and the optical fiber (rhabdom) when multiple lenses are in the vicinity of the central lens. We then derive formulas for the power absorbed in the rhabdom and for the angular sensitivity. The results of numerical simulation show that there is a less than 10 percent increase in the angular sensitivity even when five or more lenses focus light onto the rhabdom instead of a single lens. This result shows that a single lens-fiber system is a very good approximation to a multiple lens-single fiber system. This result is significant because it considerably simplifies the computations necessary to compute the inertial navigation parameters (linear, angular velocities) in the ONS (these computations are not part of this paper).

2. Model of a multiple lens, single fiber system

A schematic for the system is shown in 1(a). It consists of an effective lens that models the corneal surface and the convex lens of a single ommatidium, and an optical fiber or rhabdom that consists of photo-sensitive cells in an insect eye. The Fresnel numbers of the effective lens in a compound eye is usually very small ($\sim 1 - 10$) compared to those of mammalian eyes (~ 1000) (see [2] for a table). Due to this, the maximum of the intensity pattern between the lens and the optical fiber is not symmetric about the focal plane as can be seen in Figure 2. The entrance plane of the optical fiber is usually situated near the geometric focus of the lens for a simple apposition eye [8]. Fourier optics is important in obtaining an accurate model of the system due to (a) geometric optics fails to explain the intensity distribution near the focus, and (b) the power is propagated in the rhabdom in only a few modes. The latter fact results in loss of information, and hence in a blurry image. This smoothing effect of a lens-fiber system is taken advantage of in motion detection circuitry such as the Reichardt detector in the visual cortex of a worker bee, and in the operating principle of the ONS [2].

In Figure 1(a), R is the local radii of curvature, Θ is the inter-ommatidial angle, and they usually vary at different points on the eye [5]. There are several parameters associated with compound eyes that are of importance in a Fourier optics model. These are: (i) the F-number - defined as the ratio $\frac{f}{D_l}$ - where f is the effective focal length of the corneal surface and lens, and D_l is the facet diameter; (ii) the Fresnel number given by $\frac{D_l^2}{4\lambda f}$ where λ is the wavelength of incident light; (iii) the waveguide number (or V-number) $V = \frac{2\pi b}{\lambda}\sqrt{n_{co}^2 - n_{cl}^2}$, where b is the radius of the core, n_{co} and n_{cl} are the refractive indices of the core and the cladding. In Figure 1(b), a monochromatic, uniform plane wave excitation is incident on the lens-fiber system from the left. The plane wave is transformed by the lens and the resulting field excites modes in the fiber. The first step in determining the formula for the power propagated in the fiber is determining the electric field near the entrance of the fiber [7]. Assuming circular symmetry about the axis of the lens, the problem reduces to determining the solution to the scalar wave equation on the image side of the lens, for a monochromatic, uniform plane wave excitation. Assume that $a \gg \lambda$ and $\left(\frac{a}{f}\right)^2 \ll 1$ where a is the radius of the lens. Li and Wolf [6] give the following expression for the scalar field near the focus of a low Fresnel number lens at a point P corresponding to the polar coordinates



(a) The three lens-single fiber system.

(b) Plane wave making an angle ϕ to the axis is incident on the central lens.

Fig. 1. Schematics of the lens and optical fiber systems.

(r, ϕ, z) [6]:

$$U(P) = \frac{1}{2}\, B(u)\, \exp[i\Phi(u,v)]\,[C(u,v) - i\,S(u,v)], \tag{1}$$

where u, v, $B(\cdot)$, $\Phi(\cdot,\cdot)$, $C(\cdot,\cdot)$ and $S(\cdot,\cdot)$ are given by [6]:

$$u = 2\pi N \frac{z/f}{1+z/f}, \qquad v = 2\pi N \frac{r/a}{1+z/f}, \qquad B(u) = -\frac{2\pi i}{\lambda}\left(\frac{a}{f}\right)^2 r\left(1 - \frac{u}{2\pi N}\right)$$

$$\Phi(u,v) = \frac{1}{1-\frac{u}{2\pi N}}\left[\left(\frac{f}{a}\right)^2 u + \frac{v^2}{4\pi N}\right],$$

$$C(u,v) = \frac{\cos(u/2)}{(u/2)}\, U_1(u,v) + \frac{\sin(u/2)}{(u/2)}\, U_2(u,v)$$

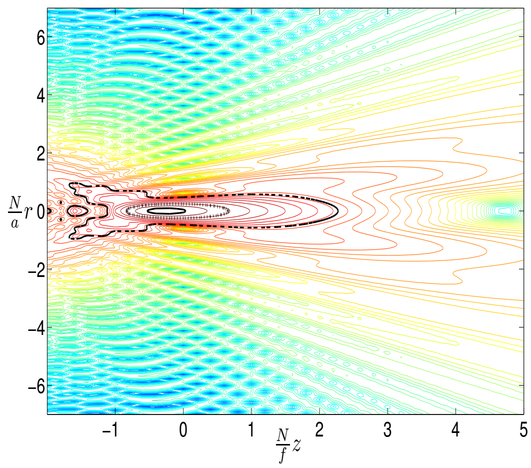
$$S(u,v) = \frac{\sin(u/2)}{(u/2)}\, U_1(u,v) - \frac{\cos(u/2)}{(u/2)}\, U_2(u,v)$$

$$U_n(u,v) = \sum_{s=0}^{\infty} (-1)^s \left(\frac{u}{v}\right)^{n+2s} J_{n+2s}(v); \quad n = 1, 2$$

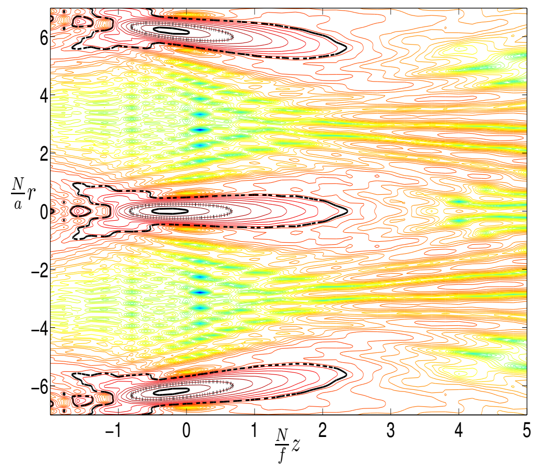
The intensity of the diffracted field at point P is simply $|U(P)|^2$ and is plotted in Figure 2(a) for a worker honey bee (*Apis Mellifera*). The parameters used were $a = \frac{D_l}{2} = 10\,\mu$m, $f = 54\,\mu$m, $\lambda = 532$ nm. For these parameter values, the Fresnel number turns out to be $N = 3.48$. The series was approximated by a partial sum with 100 terms. For the artificial eye of Jeong, Kim and Lee [4], the parameters are: $a = 18\,\mu$m, $f = 119\,\mu$m, $\lambda = 532$ nm, and the plot of the intensity is remarkably similar to 2(a). In the case of multiple lenses, we can take advantage of the fact that the principle of superposition holds for Kirchoff's integral solution to the scalar wave equation. Hence, one can add the scalar fields from multiple lenses together after converting coordinates to correspond with one another, to find the net scalar field. To be precise, consider $2M+1$ lenses (M is a natural number) arranged as shown in Figure 1(a), numbered $-M, \cdots, 0, \cdots, M$ where 0 is the central lens. Let P be a point near the focus of the central lens. Then:

$$U(P) = \sum_{j=-M}^{M} U_j(P), \tag{2}$$

where $U_j(P)$ is the scalar field (1) at the point P due to j-th lens. The plot of the intensity $|U(P)|^2$ for the worker honey bee for the case $M = 1$ is shown in Figure 2(b). We found the intensity plot for the artificial eye to be almost identical to Figure 2(b).



(a) The field near the geometric focus of a single lens.



(b) The field near the geometric focus of the central lens for a three lens system.

Fig. 2. The field near the geometric focal point of a central lens for the worker honey bee (*Apis Mellifera*). The ordinate and abscissa variables are normalized. The inner-most solid, black, bold line is .9 of the intensity, the next outer dotted line is .5, while the outermost dashed, black, bold line is .05 of the intensity. The plots for the artificial eye of Jeong, Kim and Lee [4] are almost identical to these figures.

For a monochromatic, uniform plane wave incident on a multiple lens system at an angle ϕ to the axis, we modify the steps of Barrell and Pask [1] to obtain the formula for the power propagated in the fiber corresponding to the central lens. Usually, the difference in the refractive indices of the core and the cladding is very small numerically and the fiber can be considered to be *weakly-guiding* [7]. In the weakly guiding approximation, the bound modes that propagate in the fiber are *linearly polarized* and are called LP modes (see Yariv [9], pages 66 - 78). An important parameter pertaining to the optical fiber is the waveguide number (or V-number) $V = \frac{2\pi b}{\lambda}\sqrt{n_{co}^2 - n_{cl}^2}$, where b is the radius of the core, n_{co} and n_{cl} are the refractive indices of the core and the cladding, and λ is the wavelength of the incident light. This number determines the number of modes of the electric or magnetic fields that can exist for a particular wavelength. The number of bound modes that can propagate for a step-profile fiber with waveguide number V are given by the number of real-valued solutions (U, W) to the set of equations:

$$U^2 + W^2 = V^2; \qquad U\,\frac{J_{l+1}(U)}{J_l(U)} = W\,\frac{K_{l+1}(W)}{K_l(W)} \tag{3}$$

where $J_l(U);\ l \in \mathbb{Z}$ is the l-th order Bessel's function of the first kind, and $K_l(W);\ l \in \mathbb{Z}$ is the l-th order modified Bessel's function of the second kind. For instance, for the worker bee [4]: the refractive indices are 1.363/1.340 for the core/cladding; core diameter = $2\,\mu$m. When the wavelength $\lambda = 532$ nm, there exist only two modes. On the other hand, for the artificial eye of Jeong, Kim and Lee [4]: the refractive indices are 1.614/1.584; core diameter = $5.1\,\mu$m; and there exist 14 modes for the same wavelength. For a fixed V value, the modes propagating in the fiber are given different p numbers based on the asymptotic value of the U component [7,8]. Thus, for each p, there corresponds a l-number and a pair (U, W). By means of a subscript we denote the dependence of the numbers (l, U, W) on the mode – that is, (l_p, U_p, W_p) correspond to the mode number p. The amplitude of the electric field in the fiber is given by a linear combination of fundamental modes e_p. The direction is given by the direction of the incident uniform plane wave.

$$e_p(r,\theta) = \frac{f_p(R)}{\sqrt{N_p}}\,\cos(l_p\,\theta); \qquad f_p(R) = \begin{cases} \frac{J_{l_p}(U_p\,R)}{J_{l_p}(U_p)};\ R \le 1 \\ \frac{K_{l_p}(W_p\,R)}{K_{l_p}(W_p)}\ R \ge 1. \end{cases}, \tag{4}$$

where, N_p is a normalization constant chosen so that the power propagated in each mode in the core/cladding is one (see Stavenga [8], page 7) ; R is the normalized radius $\frac{r}{b}$; b is the radius of the core.

The number of modes that propagate in the central fiber do not depend on the number of lenses considered. However, the modal coefficient a_p for mode p is the sum of coefficients $a_{p,t}$, where $t \in \{-M, \cdots, 0, \cdots, M\}$ is the index of the lens that is focusing light onto the fiber. The modal coefficient $a_{p,t}$ for a monochromatic, plane wave incident at an angle ϕ with respect to the central lens, is given by [8] (the following formula includes N_p):

$$a_{p,t}(\phi) = -\frac{2}{K\,q}\,\frac{W_p}{V}\,\sqrt{\frac{2\,n_e}{c_p\,n_i\,J_{l_p-1}(U_p)\,J_{l_p+1}(U_p)}}\,e^{j\,\Phi(H_t)}\,\int_0^{K\,q} J_{l_p}(D_t\,\Omega)G(\Omega)\,e^{-j\,\frac{H_t\,\Omega^2}{2K}}\,\Omega\,d\Omega,$$

where, $D_t = (f + h_t)\,\frac{\tan(\phi + t\,\Theta)}{b}$, where Θ is the inter-ommatidial angle; $K = \frac{2\pi\,n_i\,b}{\lambda}$; $H_t = \frac{h_t}{b}$, where $h_t = (R - f)\,(1 - \sec(t\,\Theta)) + z\,\sec(t\Theta)$ with R the local radius of curvature and z the distance of the entrance of the fiber to the geometric focus of the central lens. Other constants are: $c_p = 2$ if $l_p = 0$ and $c_p = 1$ for $l_p = 1, 2, \cdots$; $q = \frac{a\,n_i}{f}$; n_i is the refractive index of the material in the space between the lens and the optical fiber; $\Phi(H_t) = K\,H_t$; and

$$G(\Omega) = \begin{cases} \frac{V^2}{(\Omega^2 - U_p^2)\,(\Omega^2 + W_p^2)}\,(\Omega\,J_{l_p}(U_p)\,J_{l_p+1}(\Omega) - U_p\,J_{l_p}(\Omega))J_{l_p+1}(U_p) & \Omega \ne U_p \\ \frac{1}{2},\,(J_{l_p}^2(U_p) - J_{l_p-1}(U_p)\,J_{l_p+1}(U_p)) & \Omega = U_p. \end{cases}$$

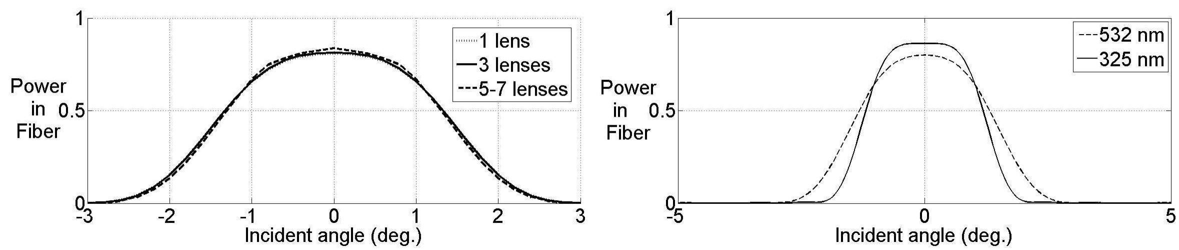
Then the power propagating in the central fiber is given by: $P = \sum_p \left|\sum_{t=-M}^{M} a_{p,t}(\phi)\right|^2$.

Figure 3 shows the result of simulations performed using the above formulae, for a worker honey bee. The parameters used for the simulations are given in Table 1. The variation of the power propagated in the central fiber as a function of the incident angle is called the *angular sensitivity* of the eye. Figure 3(a) shows the change in the angular sensitivity function for different number of lenses considered in the calculation. As the inter-ommatidial angle Θ varies in the eye of worker bee, it is worth examining the variation of the angular sensitivity with respect to Θ. The results can be seen in Figure 3(b). It can be seen that a single lens, single fiber model is a very good approximation for a worker bee.

	Worker Honey Bee	Artificial Eye		Worker Honey Bee	Artificial Eye
Lens diameter	20 μm	36μm	n_{co}	1.363	1.614
Focal length	54 μm	119 μm	n_{cl}	1.340	1.584
Core diameter	2 μm	5.1 μm	n_i	1.340	1.584
F-number	2.7	3.1			

Table 1

Eye parameters for a worker honey bee and an artificial eye of Jeong, Kim and Lee [4].



(a) Change in the angular sensitivity as a function of the number of lenses used in the model. The inter-ommatidial angle is $\Theta = 2.5^\circ$. The wavelength considered is 532 nm.

(b) Change in the angular sensitivity function with respect to inter-ommatidial angle, varied from 1° through 3°, for a three lens, single fiber system. Each of the curves above is a superposition of several curves, each for a specific inter-ommatidial angle.

Fig. 3. The power propagating in the central fiber versus the incident angle, for a unit power, monochromatic, uniform plane wave, for a worker honey bee.

3. Conclusion

In this paper, we developed a Fourier optics model of a multiple lens, single fiber system. We investigated important issues such as the change in the angular sensitivity when multiple neighboring lenses are taken into consideration, and when the inter-ommatidial angle is varied. Our simulations for a worker bee show that the neighboring lenses lead to slight increase of less than 10 percent in the angular sensitivity. There is no noticeable variation in the angular sensitivity when the inter-ommatidial angle is varied.

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