A New Control Allocation Method that accounts for Effector Dynamics

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Abstract– Given a time history of desired moments, the control allocation problem is to solve for the effector inputs so that some norm of the error between the achieved and desired moments is minimized. Existing methods solve for the actuator deflections, while accounting for Magnitude and rate limitations of the effectors. In this paper, we propose the Dynamic Control Allocation (DCA) Method, that also accounts for effector dynamics, in addition to magnitude and rate limits. We show through numerical experiments that the DCA method allocates the desired moments according to effector bandwidths - that is the slow effectors are allocated the lower frequencies in the desired moments. The numerical simulations also show that the DCA outperforms the existing simplex algorithm based LP method, that does not account for actuator dynamics.

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NOMENCLATURE

DCA	Dynamic Control Allocation
$G_{\delta,des}(t)$	3-vector of desired moments at time t
$G_{\delta}(t)$	3-vector of moments achieved by the effectors at time t
$E(t)$	Moment error $= G_{\delta,des}(t) - G_{\delta}(t)$
$\|f\|_p$	The p norm of the function f for $1 \leq p \leq \infty$
$\hat{f}$	The Fourier transform of the function f
$L^p(I)$	The space of functions defined on the interval I with bounded p norm
$J(u)$	The functional to be optimized as a function of the control input u

1. INTRODUCTION

The problems of control allocation and reconfigurable control have recently been widely studied. A review of existing methods can be found in Bodson [1]. Due to onboard, real-time computational constraints, existing methods address the problem of control allocation at each discretized time-step separately and attempt to minimize the difference between the desired and the achievable moments, while accounting for rate and magnitude limits on the effectors. This approach can be found in Buffington [2], and Doman, Ngo [3]. The downside of this approach is that the dynamics of the actuators are not taken into account, and this could mean that the moments actually achieved might be significantly different from the computed moments. This becomes a problem in reconfigurable control because the goal of reconfiguration is to recover from damaged effectors. Burken et al. [4] and Pachter et al. [5] try to include actuator dynamics by first solving an LQR problem and then solving for the closest achievable moments when rate and magnitude limits are present. We propose to solve for the inputs to the effectors in one step via an optimization procedure.

In this paper, we propose the Dynamic Control Allocation (DCA) method that takes into account individual effector dynamics as well as rate and magnitude limits. We utilize an effector model that incorporates these effects, to predict the moments achieved for some input. The DCA method minimizes the difference between the desired and "predicted" achievable moments over all possible inputs to the effector model. For effectors whose dynamics can be modeled by a linear system, our method leads to a convex optimization problem.

A different approach was taken in our earlier work [6] where we assumed the control allocation was solved by existing methods and constructed a controller for an effector with magnitude-limited first-order dynamics so that it can follow the commanded deflection. However, the analysis gets quite complicated for effectors with higher-order dynamics, while the method presented here is less complex.

Mathematical Preliminaries

Let $I = [t_0, t_1]$ be an interval of time over which the control allocation problem is required to be solved. Let

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$G_{\delta,des}(t)$ denote the 3-dimensional vector of desired moments and $\delta(t)$ denote the n-dimensional vector of control effector positions at time $t \in I$. We assume $n \geq 3$. The space of functions $G_\delta(t)$ and $G_{\delta,des}(t)$ where $t \in I$, is a vector space on which we can consider several types of norms. The useful norms that could be considered are defined as follows:

$$\|f\|_p = \left(\int_I |f(t)|^p \, dt\right)^{\frac{1}{p}} \quad (1)$$

$$\|f\|_\infty = \operatorname*{ess\,sup}_{t\in I} |f(t)| \quad (2)$$

If $\|f\|_p < \infty$ for some p such that $1 \leq p \leq \infty$ then f is said to be in the normed vector space $L^p(I)$.

The control allocation problem can be formulated in the time-domain in terms of the moment-error, or in the frequency domain by considering the Fourier transform of moment-error. The Fourier transform of a function $f \in L^1(I)$ is the function $\hat{f}$ defined by:

$$\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_I f(t) e^{-j\omega t} dt, \quad (3)$$

where $j = \sqrt{-1}$. Then we have the following useful theorem:

Theorem 1.1: (Rudin [7])

- If $f \in L^1(I)$, then $\hat{f} \in C_0(\mathbb{R})$, and $\|\hat{f}\|_\infty \leq \|f\|_1$.
- If $f \in L^2(I)$, then $\|\hat{f}\|_2 = \|f\|_2$ (Parseval-Plancheral).

Here $C_0(\mathbb{R})$ is the supremum-normed Banach space of all complex continuous functions on $\mathbb{R}$ that vanish at infinity. The Fourier transform of a vector-valued function is defined component-wise. If $E(t) = [E_1(t) \; E_2(t) \; E_3(t)]^T$, then $\hat{E}(j\omega) = [\hat{E}_1(j\omega) \; \hat{E}_2(j\omega) \; \hat{E}_3(j\omega)]^T$. If $E_i(t)$, $i = 1, 2, 3 \in L^1(I)$ or $L^2(I)$, then Theorem 1.1 can be applied as follows:

$$\|\hat{E}(\cdot)\|_\infty := \max_{i=1,2,3} \|\hat{E}_i(\cdot)\|_\infty \quad (4)$$

$$\leq \max_{i=1,2,3} \|E_i(\cdot)\|_1 \quad (5)$$

$$\leq \sum_{i=1}^{3} \|E_i(\cdot)\|_1 \quad (6)$$

$$= \|E(\cdot)\|_1; \quad (7)$$

and

$$\|\hat{E}(\cdot)\|_2 := \left(\sum_{i=1}^{3} \|\hat{E}_i(\cdot)\|_2^2\right)^{\frac{1}{2}} \quad (8)$$

$$= \left(\sum_{i=1}^{3} \|E_i(\cdot)\|_2^2\right)^{\frac{1}{2}} \quad (9)$$

$$= \|E(\cdot)\|_2. \quad (10)$$

In the control allocation problem, $E(t)$ is taken to be the difference between the desired moments and the achieved moments.

2. Control Allocation

Suppose there are n control effectors. The effectors are assumed to satisfy the differential equations:

$$\dot{x}_i(t) = f_i(x_i(t), u_i(t)), \quad (11)$$

$$\delta_i(t) = h_i(x_i(t), u_i(t)) \quad i = 1, \cdots n, \quad t \in I \quad (12)$$

The vector functions f_i and h_i could be nonlinear due to rate and/or magnitude limitations. The ***control effectiveness function*** $B(p(t), \cdot)$ maps $\delta(t) = [\delta_1(t) \cdots, \delta_n(t)]^T$ to the 3-dimensional vector of moments $G_\delta(t)$ produced by the effectors:

$$G_\delta(t) = B(p(t), \delta(t)); \quad t \in I, \quad (13)$$

where $p(t)$ is a set of parameters such as Mach number, angle of attack and side slip angle. It is assumed in this work that the time scale on which $B(p(\cdot), \cdot)$ changes is much larger than the length of the interval I. This assumptions is justified because the parameters $p(t)$ are typically slowly changing. Thus we regard $B(p(t), \cdot)$ to be denoted by $B(\cdot)$, so that:

$$G_\delta(t) = B(\delta(t)); \quad t \in I. \quad (14)$$

Note that this general definition can also be used to model actuator interactions. Define $E(\cdot) = G_{\delta,des}(\cdot) - G_\delta(\cdot)$ to be difference between the desired and achieved moments. As usual, we denote $\hat{E}(\omega)$ as the Fourier transform of $E(t)$.

The control allocation problem can be posed in several ways:

Problem Statement 1: Obtain the control inputs to the effectors $u_i(t)$; $i = 1, \cdots n$, $t \in I$, so that $\|E(\cdot)\|_p$ is minimized for some p such that $1 \leq p \leq \infty$.

Problem Statement 2: Obtain the control inputs to the effectors $u_i(t)$; $i = 1, \cdots n$, $t \in I$, so that $\|\hat{E}(\cdot)\|_q$ is minimized for some q such that $1 \leq q \leq \infty$.

To begin the development of the DCA, denote $J_p(u) = \|E(\cdot)\|_p$ and $\tilde{J}_q(u) = \|\hat{E}(\cdot)\|_q$. Also denote $u_p = \arg\min_{u\in L^1(I)} J_p(u)$ and $\tilde{u}_p = \arg\min_{u\in L^1(I)} \tilde{J}_p(u)$. Then it is clear from the discussion in the mathematical preliminaries that:

- $\|\hat{E}\|_\infty \leq \|E\|_1 \implies \min_u \tilde{J}_\infty(u) \leq \min_u J_1(u)$;
- $\|\hat{E}\|_2 = \|E\|_2 \implies \min_u \tilde{J}_2(u) = \min_u J_2(u)$.

From the second result, we see that the same solution u_2 minimizes the 2-norm of the moment-error or its Fourier transform. The same is true when $\min_u J_1(u) = 0$ (or equivalently, $\min_u \tilde{J}_\infty(u) = 0$). Constraints on the set in which u belongs could lead to $\min_u J_1(u) \neq 0$. By our notation, $u_1 = \arg\min_{u \in L^1(I)} J_1(u)$, and it is possible that there exists $u_\infty = \arg\min_{u \in L^1(I)} \tilde{J}_\infty(u)$ that satisfies $\tilde{J}_\infty(u_\infty) < \tilde{J}_\infty(u_1)$. If our goal is to allocate controls according to bandwidth of the actuators, then it seems that a more natural cost function would be $\tilde{J}_\infty(u)$, rather than $J_1(u)$. *We call the solution u_∞ as the optimal solution.* However, as the minimization problem is more easily solved in the time-domain (which leads to the solution u_1), we obtain a *sub-optimal* solution. The 1-norm is used frequently in Control allocation literature as it leads to linear programming approaches to the numerical solution [1], [2], [3].

Existing methods allocate control at a time instant t by minimizing the moment-error at that time alone [1], [2], [4], [5], [3]. One can see that as the interval I in our discussion collapses to one point, we obtain the existing methods. Some methods take into account actuator rate and magnitude limits in the control allocation problem but do not include actuator dynamics [1], [2], [3]. Other methods try to include actuator dynamics by first solving an LQR problem and then solving for the closest achievable moments (using LP or QP) when rate and magnitude limits are considered [4], [5].

Our proposed method includes both actuator dynamics as well as rate and magnitude limits on the actuators. Suppose that the Equations 11 are integrated to yield:

$$x_i(t) = \int_{t_0}^{t} f_i(x_i, u_i)\, dt \tag{15}$$

$$\delta_i(t) = h_i(x_i(t), u_i(t)) \quad i = 1, \cdots, n \tag{16}$$

where $t \in I$. We consider the "current" time to be in I and the problem is to solve for $u(t)$ for the entire interval I. Once this is done, the inputs corresponding to the current time is applied to the actuators.

We discretize the time axis into instants $\tau_k,\ k = 0, \cdots, K$ such that $\tau_0 = t_0$ and $\tau_K = t_1$. The problem is then:

$$\min_u J(u) = \min_u \|E(\cdot)\|_p \tag{17}$$

$$= \min_u \|B(\delta(\cdot)) - G_{\delta,des}(\cdot)\|_p, \tag{18}$$

where $p = 1$ or 2. There is no constraint on the set to which u belongs. When the system dynamics are linear, then $\delta(\cdot)$ is a linear function of $u(\cdot)$. Furthermore, if the achieved moments $B(\delta(\cdot))$ can be approximated by a linear function $B_1\,\delta(\cdot)$ where B_1 is a $3 \times n$ matrix, then $J(u)$ is a convex functional of the input function $u(\cdot)$. The linear approximation of the control effectiveness function is valid when I is a small enough interval and can be seen in works of other researchers [1], [2], [3].

3. Numerical Experiments

We consider control effectors such as flaperons, rudders only in the following numerical study. We assume the air vehicle is on an unpowered descent so that the engine is not used, though it should be noted that our method is general enough to handle complex engine models also. Each effector is considered to be a 2nd order linear system with magnitude and rate limitations as shown in Figure 1. In the figure, the signals δ^j_{des} correspond to u_j in Equations 15 and 16. The constants for the actuators are shown in Table 1.

The control effectiveness matrix was taken to be:

$$B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.5 & 0.5 & 1 & 1 \\ 2 & 2 & 1 & 1 \end{bmatrix}. \tag{19}$$

The numerical experiments were conducted with t_0 equal to the current time and the interval I only included the current time, so as to allow a comparison with existing methods. The time-axis discretization used was 0.02 seconds. The minimization of the cost function 18 with $p = 2$ was performed with the 'ucSolve' routine in TOMLAB. The constants K_1 and K_2 in Figure 1 are computed using the values for the natural frequency ω and damping coefficient ζ in Table 1 according to: $K_1 = 2\zeta\omega$ and $K_2 = \omega^2$.

Figure 2 shows the result of the numerical experiments with $I = \{t_0\}$. The desired moments $G_{\delta,des}(\cdot)$ were taken to be $2\sin(\frac{5t^2}{t_f})[1\ 1\ 1]^T + [3.2\ 2.4\ 4.8]^T$, where $t_f = 9$. Figure 2(a) shows the desired moments and the moments achieved by our proposed Dynamic Control Allocation (DCA) method. It also shows the moments commanded and achieved by the simplex method based LP algorithm in Bodson [1]. For the latter method, the commanded deflections δ_{des} were computed only considering the rate and magnitude limits on the effectors and ignoring the dynamics. The commanded moments in Figure 2(a) are then given by $B\,\delta_{des}$. Then the commanded deflections are applied to a model of the actuators and the achieved moments are computed according to $B\,\delta$. One can see that though $B\,\delta_{des}$ tracks the desired moments well, the achieved moments are far from satisfactory. On the other hand, the DCA method produces achieved moments that track the desired moments better.

Figure 2(b) shows the commanded and response deflections of the effectors. One can clearly see that initially the slow changing moments are allocated to the rudders (with the slow dynamics). As time increases, the rudders prove incapable of tracking the faster changing signals and the flaps take up the slack at an increasing rate. *Thus both optimal-tracking and allocation of moments according to the bandwidth of the actuators are achieved simultaneously.* One

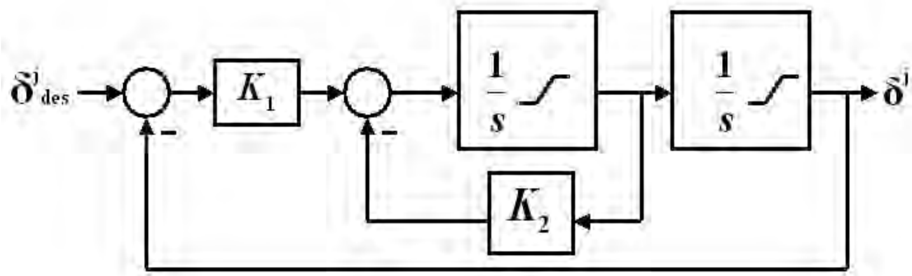

Figure 1. Effector schematic diagram

	Natural Freq. rad/sec	Damping Coeff.	Rate Limit Deg/sec	Magnitude Limit (low,high) Deg
Flap 1	15	0.75	4	(0, 1.5)
Flap 2	15	0.75	4	(0, 1.5)
Rudder 1	5	0.75	2	(0, 1.5)
Rudder 2	5	0.75	2	(0, 1.5)

Table 1. Actuator parameters for Figure 2.

can also see that the commanded signals δ_{des} violate both the rate and magnitude limits as opposed to the existing LP method; however, the actual rate and position limits are satisfied by δ.

4. Conclusion

In this paper, we proposed a new control allocation methodology that takes into account rate and magnitude limits on the effectors, as well as their dynamics. Existing methods only take into account rate and magnitude limits. We solve for the inputs to the effectors so that the achieved moments match the desired moments over an interval of time. One of the interesting results obtained was that of frequency separation - the slower effectors were allocated the lower frequencies in the desired signal while the faster effectors were allocated the higher frequencies.

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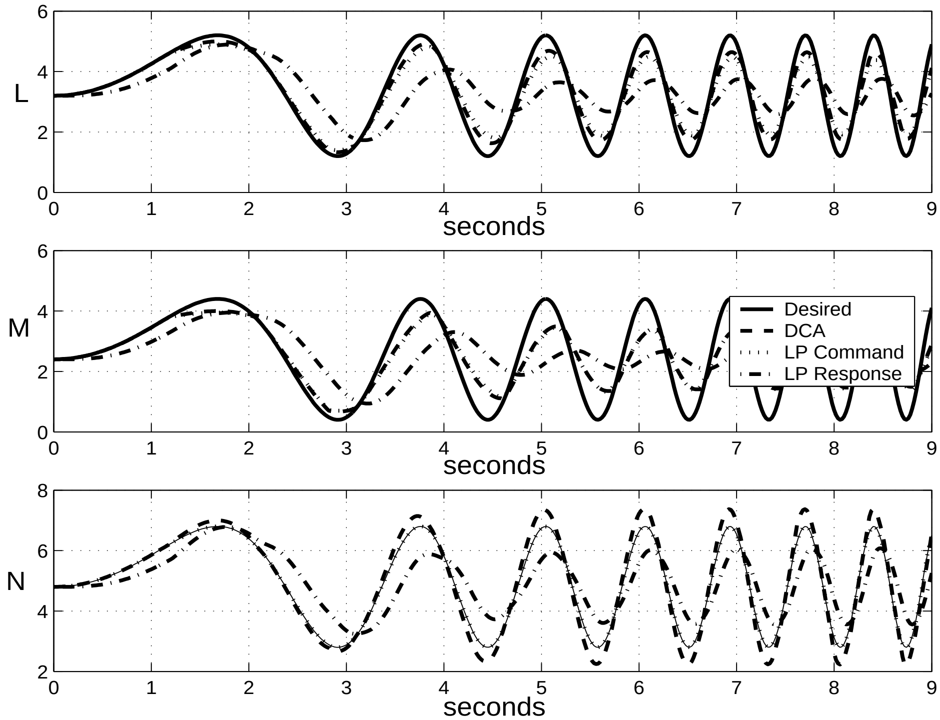
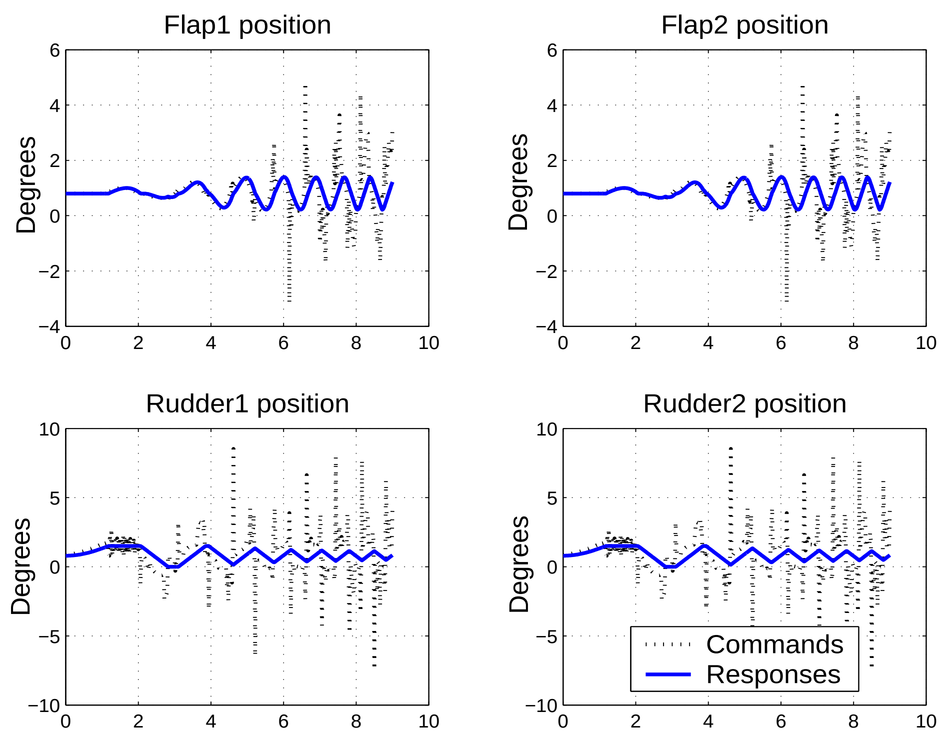


(a) Desired versus achieved moments for DCA and LP schemes.



(b) Effector command and response signals with DCA scheme.

Figure 2. Control allocation results with DCA and LP schemes.