1. What conditions on $a$ and $b$ guarantee that $z^a(1 - z)^b$ can be defined as a single-valued function on $\mathbb{C}\setminus[0, 1]$? In this case describe the Riemann surface of this function?

2. Show that the set of points $\{(z, w) \in \mathbb{C}^2 | w^2 = \sin z\}$ is a Riemann surface.

3. Let $P(z)$ be a polynomial with $2n + 1$ distinct complex roots. Show that the compactification of $X = \{(z, w) | w^2 = P(z)\}$ is a Riemann surface (this is called a hyperelliptic surface). What is its genus?

4. With the notation from the lecture notes, show that for every $\omega_1$ and $\omega_2$ such that $\text{Im}\frac{\omega_2}{\omega_1} > 0$, there is exactly one $\tau$ in the upper half plane such $\mathbb{C}/\Lambda$ and $\mathbb{C}/L$ are conformally equivalent.