1. Show that any biholomorphic map from the upper half-plane onto the open unit disk has the form

\[ f(z) = e^{i\theta} \frac{z-a}{z-\bar{a}}. \]

2. Let \( f \) be the Riemann map of a simply connected domain \( D \) onto the open unit disk satisfying \( f(z_0) = 0, f'(z_0) = A > 0 \). Show that if \( g \) is an analytic function on \( D \) satisfying \( |g| < 1 \), then \( |g'(z_0)| \leq A \), with equality if and only if \( f = e^{i\theta} g, \theta \in \mathbb{R} \).

3. Let \( f(z) \) be analytic for \( 0 < |z| < 1 \), and define \( f_n(z) = f(z/n) \) for \( 0 < |z| < 1, n \geq 1 \). Show that \( \{f_n\} \) is a normal family on the punctured disk if and only if the singularity of \( f \) at 0 is a pole or removable.

4. Find a biholomorphic map from \( \mathbb{C}\setminus(-\infty,0] \) to \( D = \{z \mid |z| < 1\} \).

5. Find a biholomorphic map from the domain

\[ \{z = re^{i\theta} \mid r \in \mathbb{R}, \ 0 < \theta < 2\pi/3\} \]

to the upper half plane \( \text{Im} \ z > 0 \).

6. Show that \( w(z) = \frac{1}{2}(z + \frac{1}{z}) \) maps the interior of the unit disk conformally onto a domain \( D \) on the Riemann sphere. What is \( D \)? What is the inverse map?