Complex Analysis – Homework 1

1. Find the radius of convergence of $\sin z$ and $\cos z$.

2. Find the radius of convergence for the following power series

$$\sum_{n=0}^{\infty} n^2 z^n, \quad \sum_{n=0}^{\infty} \frac{2^n z^{2n}}{n^2 + n}.$$

3. Find the radius of convergence of the power series that is the Taylor series of

$$f(z) = \frac{2z + 3}{4z + 5}$$

at the origin.

4. Find the image of the sets

\{z \mid \text{Re} z < 0, |\text{Im} z| < \pi\}
\{z \mid |\text{Im} z| < \pi/2\}

under the exponential map.

5. Discuss the mapping properties of $\cos z$, $\sin z$, $z^n$, $n \geq 2$.

6. Find the Möbius transformations that map the unit disk onto itself.

7. Find a holomorphic map that maps \{z \mid \text{Re} z > \text{Im} z\} to \{z \mid |z-1| < 2\}. 