Homework 6

1. Prove that for every real number $x$, $e^x \geq x + 1$.

2. Find all positive real solutions to the equation $2^x = x^2$.

3. Show that for every positive numbers $a, b$, and positive integer $n$, one has $a^n + (n - 1)b^n \geq nab^{n-1}$.

4. Compute the following limits using l’Hospital’s theorem:

$$
\lim_{x \to 0} \frac{1 - \cos x}{x^2} \\
\lim_{x \to \infty} \frac{\ln x}{x^2} \\
\lim_{x \to 0^+} x \ln \sin x \\
\lim_{x \to 0^+} x^x.
$$