Homework 1

1. Prove that
\[ \lim_{n \to \infty} \frac{2n^2 + 3}{n^2} = 2. \]

2. Guess the limit
\[ \lim_{n \to \infty} \frac{n + 3}{2n + 1}, \]
then prove it.

3. Guess the limit of the sequence \( x_n = \frac{1}{2^n}, \ n \geq 1 \), then prove that this is true.

4. Prove that
\[ \lim_{n \to \infty} \frac{n^2 + n}{n^2} = 1. \]

5. Prove that
\[ \lim_{n \to \infty} \sqrt{\frac{n + 1}{4n}} = \frac{1}{2}. \]