Homework 1

In all of the following problems, you are supposed to use only the definition of convergence.

1. Prove that the sequence $x_n = \frac{1}{3^n}$, $n \geq 1$ converges to zero.

2. Prove that the sequence $x_n = \frac{n+(-1)^n}{n+2}$, $n \geq 1$, converges and find its limit.

3. Prove that the Fibonacci sequence $F_1 = 1$, $F_2 = 2$, $F_{n+1} = F_n + F_{n-1}$ does not converge.

4. Prove that
   \[ \lim_{n \to \infty} \frac{3n^2 + 1}{2n^2 + 2} = \frac{3}{2} \]

5. Show that if $x_n \geq 0$ for all $n \geq 1$ and $\lim_{n \to \infty} x_n = 0$, then $\lim_{n \to \infty} \sqrt{x_n} = 0$.

6. Show that the sequence
   \[ \frac{n^2 + 2n + 1}{n^3 + 3n^2 + 3n + 1} \]
   converges and find its limit.

7. Show that if the sequence $(x_n)$ converges, then so does the sequence $(|x_n|)$.

8. Does the sequence
   \[ x_n = \frac{\sqrt{n}}{n+1}, \quad n \geq 1 \]
   converge?

9. Prove that the sequence $(2^n/n!)$ converges, and find its limit.

10. Prove that
    \[ \lim_{n \to \infty} \sqrt{n} = 1. \]